Journal

of

THE ROYAL AERONAUTICAL SOCIETY

OCTOBER 1957

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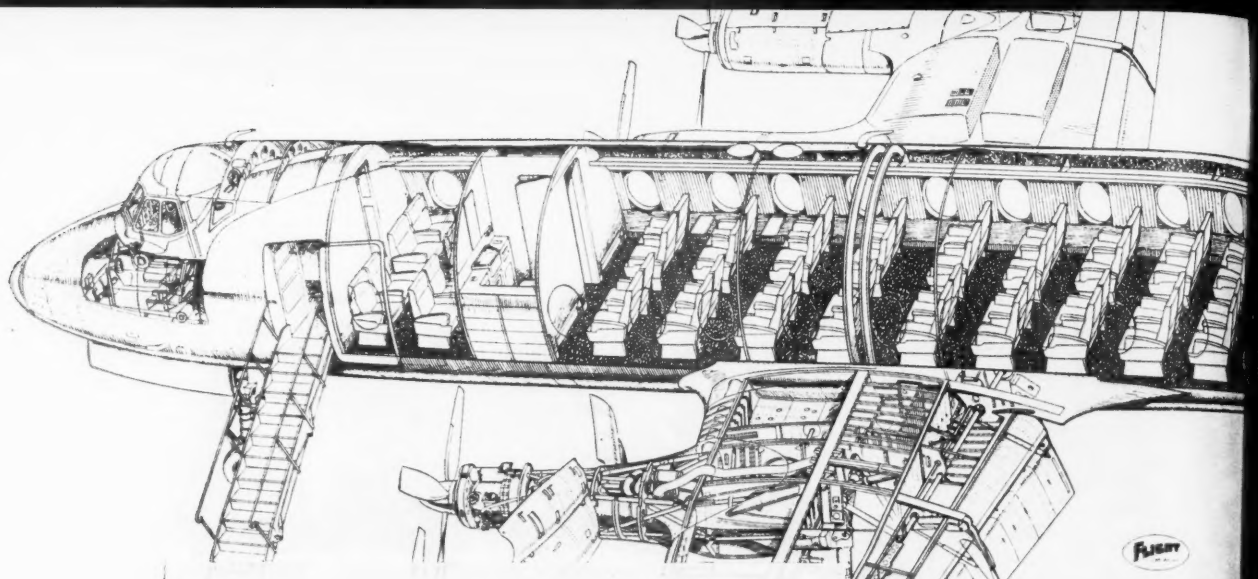

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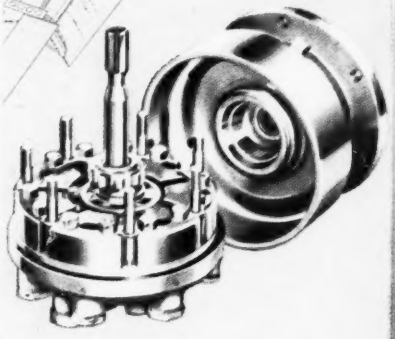

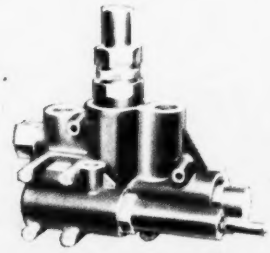

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PUBLISHED MONTHLY AT

4 HAMILTON PLACE, LONDON, W.1

Telephone: Grosvenor 3515-9 *Telegrams:* Didaskalos, Audley, London

SUBSCRIPTIONS: £7 10s. 0d. PER ANNUM, PLUS 9s. POSTAGE AND PACKING.
SINGLE COPIES: 12s. 6d. PLUS 9d. POSTAGE AND PACKING.

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All communications for publication in, and advertisements for, the JOURNAL *should be addressed to the Editor,* JOAN BRADBROOKE, *A.R.Ae.S.; all communications on general matters affecting the Society should be addressed to the Secretary,* A. M. BALLANTYNE, *T.D., B.Sc., Ph.D., Hon.F.C.A.I., A.F.I.A.S., F.R.Ae.S.*

None of the papers or paragraphs must be taken as expressing the opinion of the Council of the Royal Aeronautical Society unless otherwise stated.

PRINTED BY THE LEWES PRESS, WIGHTMAN & CO. LTD., LEWES, SUSSEX, ENGLAND, AND PUBLISHED BY THE ROYAL AERONAUTICAL SOCIETY, 4 HAMILTON PLACE, LONDON, W.1, ENGLAND

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NOTICES

Royal Aeronautical Society Scholarships and Grants 1957/58

The following Scholarships and Educational Grants have been awarded by the Council:—

The Edward Busk Studentship in Aeronautics—to D. J. Maull of 13 Gardner Road, Guildford, Surrey, for research work in Aeronautics at Imperial College. He is 24 years old and holds the degree of B.Sc.(Eng.) of London University.

The Geoffrey de Havilland Memorial Scholarship—to J. Counihan, of 19 Kingston Hill, Kingston-on-Thames, Surrey, for post-graduate work at the College of Aeronautics, Cranfield, in Aerodynamics. He is 23 years old and holds the Higher National Diploma in Mechanical Engineering.

Educational Grant—to A. L. Yettram, of 17 Wyburn Avenue, Barnet, Herts, to enable him to continue his studies in aircraft structures for a Diploma of Imperial College. He is 24 years old and holds the B.A. and the B.A.I. (Bachelor of Engineering) degrees, both of Trinity College, Dublin University.

Educational Grant—to G. E. Cook, of 70 Wallace Road, Coventry, to enable him to continue his theoretical work on flow in intakes at transonic speeds at Imperial College. He is 22 years old and was awarded the Charter Scholarship in 1956. Mr. Cook holds the degree of B.Sc.(Eng.) of London University.

Educational Grant—to T. J. Black, of 4 Kimberley Road, Cambridge, to enable him to continue his work for his thesis on Boundary Layer Control for High Lift. He is 23 years old and was awarded the Charter Scholarship in 1955. Mr. Black holds the degree of B.Sc. of Belfast University and won a Whitworth Prize in 1953 and a Whitworth Scholarship in 1954. He also received an Educational Grant in 1956.

Educational Grant—to J. I. Dodds, of 348 Chanterlands Avenue, Hull, to enable him to continue his research into the study of the Turbulent Boundary Layer. He is 23 years old and holds a B.A. degree in Mechanical Science of Cambridge University. Mr. Dodds was awarded the Edward Busk Studentship in Aeronautics in 1955 and an Educational Grant in 1956.

Educational Grant—to I. M. Allison, of 2 Holland Road, Withycombe, nr. Exmouth, Devon, to enable him to continue his photoelastic work on stress concentration factors in shafts and tubes at University College, London. He is 23 years old and holds the B.Sc. degree in Mechanical Engineering.

(The Charter Scholarship was not awarded.)

British Commonwealth Lecture—Postponed

The Thirteenth British Commonwealth Lecture, which was to have been given on 17th October 1957, has been postponed until *March 1958*. The provisional date is Thursday 27th March but this will be confirmed later when the venue is known. The Lecture will be by Mr. B. S. Shenstone, M.A.Sc., B.A.Sc., F.C.A.I., A.F.I.A.S., F.R.Ae.S., on "Why Airlines are Hard to Please."

Guided Flight Section

The first lecture of the Guided Flight Section of the Society will be given on Thursday 21st November by Mr. J. E. Serby, C.B.E., F.R.Ae.S., Director-General, Guided Weapons, Ministry of Supply, at the Royal Institution, Albemarle Street, W.1, at 6.00 p.m. Tea will be served at 5.30 p.m.

The lecture will be on "Guided Weapons and Aircraft —Some Differences in Design and Development."

Lost—and Found

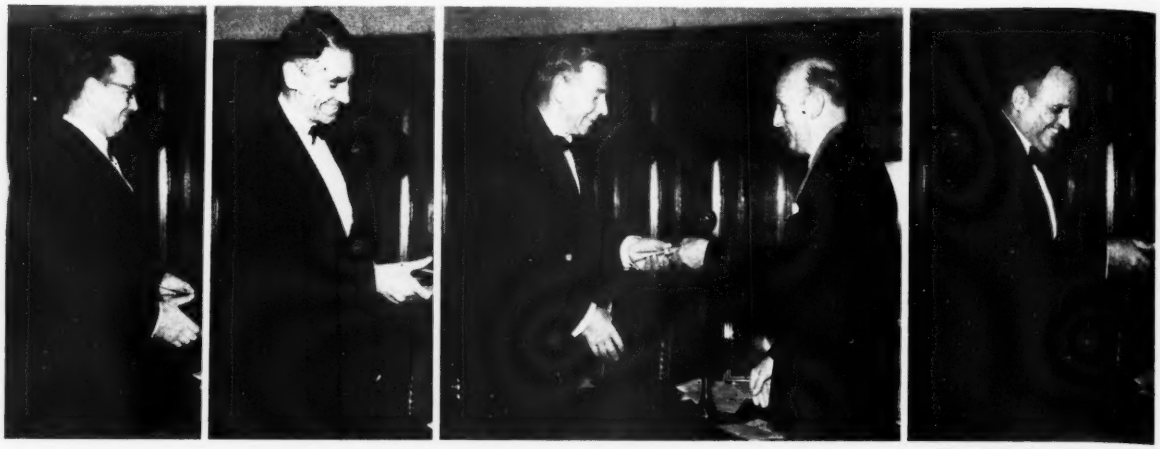
Folkestone—*The Mayor's Reception, 11th September* 1957

Will the member who was inadvertently given a very new, nylon-covered umbrella with gold band, in mistake for his own similar but not so new one, by the cloakroom attendant at the Hotel Metropole, Folkestone, after the Mayor's Reception on 11th September, kindly inform the Society's offices so that the umbrellas can be exchanged.

Garden Party—15th September 1957—A camera lens hood, 3·2 cm. diameter fitting, was found near the static aircraft display at the Garden Party at Wisley on Sunday 15th September, and may be claimed from the offices of the Society.

Photographs by courtesy of Flight

As has now become customary, the main awards of the Society were presented on the occasion of the Wilbur Wright Memorial Lecture on 13th September. The photograph on the left shows H.R.H. The Prince of The Netherlands, who has honoured the Society by accepting Honorary Fellowship, receiving the Scroll of Honorary Fellowship from the President, Sir George Edwards. On the right Miss B. Voyce who resigned last year after many years on the staff of the Society and who has been made an Honorary Companion of the Society.



Photographs by courtesy of Flight

Medals presented by the President, Sir George Edwards, C.B.E., F.R.Ae.S., at the 45th Wilbur Wright Memorial Lecture on 13th September included, from left to right:— The Society's Silver Medal to Mr. M. B. Morgan, M.A., F.R.Ae.S., Deputy Director, Royal Aircraft Establishment for "his contributions to the execution and direction of aeronautical research and development." The Society's Bronze Medal to Mr. F. B. Greatrex, B.A., F.R.Ae.S., Manager, Rolls-Royce, Hucknall for "his work on the reduction of noise from aero-engines." The British Gold Medal to Mr. R. L. Lickley, B.Sc., F.R.Ae.S., Technical Director and Chief Engineer, Fairey Aviation Co. Ltd., for "his outstanding practical contributions to aircraft design and development." The British Silver Medal to Mr. Charles Abell, O.B.E., F.R.Ae.S., Chief Engineer, B.O.A.C. for "his engineering achievements contributing to efficiency in airline operations."

News of Members

R. D. Anthony (*Associate Fellow*), formerly Section Leader with Vickers-Armstrongs (Aircraft) Ltd., Supermarine Works, has joined the Nuclear Power Plant Co., Nutsford, as Mechanical Engineer.

F. Bradshaw (*Associate Fellow*) is to take up an appointment in the Winnipeg Branch of his company, Bristol Aircraft Ltd.

N. Bull (*Associate*), formerly Deputy Chief Inspector at R.N.A.Y. Donibristle, has now been appointed Chief Inspection Officer at R.N.S.D. Llangennech.

Lieut. W. A. Caws (*Associate Fellow*), formerly in H.M.A.S. Albatross, is now an Engineer Officer, H.M.A.S. Voyages, N.S.W., Australia.

E. H. Crudden (*Associate*), formerly at Bristol Aircraft Ltd., is now a Senior Design Engineer with the Lockheed Aircraft Corp., Marietta, Georgia.

H. St. L. Dannatt (*Associate Fellow*), formerly with Sperry Gyroscope Co., Brentford, is now employed as an Engineer in the Special Weapons Division of Canadair Ltd., Montreal.

W. G. FitzGibbon (*Associate*), formerly of de Havilland Aircraft Co. Ltd., is now with Canadair Ltd. in Montreal.

Sqdn. Ldr. G. G. Fowler (*Associate Fellow*), formerly of Flag Officer Reserve Aircraft, R.N.A.S. Arbroath, is to be O.C. Engineering Squadron, R.A.F., Changi, Singapore.

A. Frenkel (*Associate Fellow*) has relinquished his post as Chief Engineer with Field Aircraft Services to become Senior Designer to Bristol Aircraft (Western) Ltd., Winnipeg.

K. T. Fulton (*Graduate*), formerly on the editorial staff of "The Aeroplane," is now a Technical Assistant in the Business Office of the de Havilland Engine Co. Ltd. at Leavesden.

R. Hickling (*Graduate*) a Scientific Officer with the Admiralty has been awarded a year's fellowship at the California Institute of Technology and has been granted leave from the Admiralty.

L. W. Kilbourne (*Associate*), formerly of Rolls-Royce Ltd., is now Technical Engineer with Dalton and Co. Ltd., Oil Refiners of Belper, Derbyshire.

J. C. King (*Associate Fellow*) of English Electric Co. Ltd. is to take up a position as an Assistant Director of Engineering at Marconi House.

Flight Lieut. N. E. C. Maggs (*Associate*) has retired from the Royal Air Force and is now Technical Assistant at Westland Aircraft Ltd., Yeovil.

Lt. Cmdr. M. C. J. Major (*Associate*), formerly with Portsmouth Aviation Ltd., is now on the Sales Service Staff of Canadair Ltd., Montreal.

N. V. McEachern (*Student*), formerly of Canadair Ltd., Dynamics Section, is now with the Low Speed Aerodynamics Laboratory of the National Research Council, Ottawa.

Captain G. L. Newton-Wade (*Associate*), formerly R.E.M.E., is to take up an appointment in charge of Technical Services Branch, Army Air Corps.

Sqdn. Ldr. R. C. Rogers (*Associate Fellow*) has completed the two-year Cranfield Diploma Course and has taken up an appointment in the Directorate of Fighter Aircraft Research and Development, M.O.S.

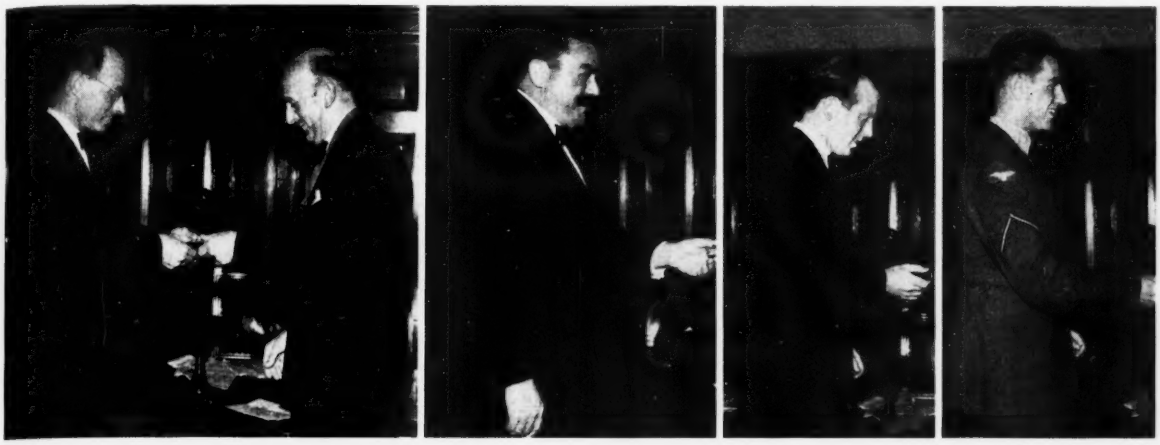
E. H. Smith (*Graduate*) has recently been appointed Chief Technician at Baynes Aircraft Interiors Ltd., Langley, Bucks.

Fred W. Stockhausen (*Associate*) has resigned his position as Chief Inspector with Steel Improvement and Forge Co., Ohio, and has taken up a similar position with Lake Shore Industries Inc., Batesburg, S.C.

Wing Cmdr. J. P. Thompson (*Associate Fellow*) an Associate of Messrs. Norman and Dawbarn, formerly resident in Jamaica, has now returned to England.

F. Williams (*Associate Fellow*), formerly Senior Technical Assistant, D. Napier and Son Ltd., is now employed by English Electric Co. Ltd., Gas Turbine Dept., Whetstone, as Technical Engineer (Aerodynamics).

Flight Lieut. B. N. Wilson (*Associate Fellow*), formerly lecturer in the Electrical Wing (Engineering) of the R.A.F. Henlow, is now Electrical and Instrument Officer at R.A.F. Khormaksar (M.E.A.F. Aden).



Photographs by courtesy of Flight

From left to right:— Mr. Caradoc Williams, B.Sc., M.I.E.E., Assistant Director, Engineering Division of the Radio Department, R.A.E., receiving the Wakefield Gold Medal for "his contributions to the development and use of radio aids to navigation." Mr. Bernard Lynch, B.E.M., Chief Instructor, Martin-Baker Aircraft Co. Ltd., receiving the R. P. Alston Medal for "his practical achievement in the flight testing of ejector seats"; originally founded as a prize in 1940 this is the first time this award has taken the form of a Medal. Other new Medals presented for the first time were the N. E. Rowe Medals founded in 1956 for two age groups between 21 and 26 years and, under 21, to encourage the younger Members of the Branches. The last two photographs show Mr. P. A. Sutherby, B.A., now with Elliott Bros. (London) Ltd. receiving the N. E. Rowe Medal for his lecture "Possible Flight Paths for Helicopters," given to the Bristol Branch and Mr. R. D. Trender, Junior Technician, R.A.F., receiving the second N.E. Rowe Medal for his paper "The History and Development of Parachutes and British Ejector Seats" given to the Halton Branch.

DIARY

LONDON

8th October

SECTION LECTURE.—Some Thoughts on Non-Linear Problems in Aerodynamics. P. J. Dunctон. Library, 4 Hamilton Place, London, W.1. 7 p.m.

22nd October

SECTION LECTURE.—Aircraft Instrumentation for Flight Test Purposes. R. R. Duddy and I. McLaren. Library, 4 Hamilton Place, London, W.1. 7 p.m.

12th November

SECTION LECTURE.—Some Problems of Stability and Control. H. H. B. M. Thomas. Library, 4 Hamilton Place, London W.1. 7 p.m.

19th November

SECTION LECTURE.—The Supersonic Wind Tunnel. Dr. W. F. Hilton. Library, 4 Hamilton Place, London, W.1. 7 p.m.

26th November

SECTION LECTURE.—Some Aspects of Refrigeration in Supersonic Aircraft. E. J. Gabbay. Library, 4 Hamilton Place, London, W.1. 7 p.m.

GUIDED FLIGHT SECTION

21st November

GUIDED WEAPONS AND AIRCRAFT.—Some Differences in Design and Development. J. E. Serby. The Royal Institution, Albemarle Street. 6 p.m. (Tea at 5.30 p.m.)

GRADUATES' AND STUDENTS' SECTION

16th October

Trends in Air Transport. P. W. Brooks. Library, 4 Hamilton Place, London, W.1. 7.30 p.m.

7th November

The Background to Aircraft Specifications. K. W. Clark. Library, 4 Hamilton Place, London, W.1. 7.30 p.m.

27th November

Flight-testing Procedures at Supersonic Speeds. Wing. Cdr. R. P. Beamont. Library, 4 Hamilton Place, London, W.1. 7.30 p.m.

29th November

Annual Dance.—4 Hamilton Place, London, W.1.

BRANCHES

1st October

Boscombe Down.—The Study of the Upper Atmosphere Using Rockets. Dr. E. B. Dorling, Ph.D. Lecture Hall, A. and A.E.E., Boscombe Down. 5.45 p.m.

7th October

Halton.—Lecture by representative of Bomber Command. Branch Hut, R.A.F. Halton. 6.45 p.m.

8th October

London Airport.—Some Aspects of the Development of the Fokker Friendship. H. Van Meerton. Staff Restaurant, B.O.A.C., London Airport. 6.0 p.m.

Luton.—Human Limitations of High Performance Flight. Sqdn. Ldr. T. C. D. Whiteside. Napier Senior Staff Canteen, Luton Airport. 6.15 p.m.

9th October

Brough. Recent Developments in Production Processes. W. E. Goff. Lecture Hall, Electricity Offices, Ferensway, Hull. 7.30 p.m.

Chester.—Gliding. F. N. Slingsby. Lecture Theatre, Grosvenor Museum, Chester. 7.30 p.m.

Christchurch.—Ticket to Australia. R. N. Hollock. King's Arms Hotel, Christchurch. 7.30 p.m.

12th October

Halton.—Annual General Meeting, 4.0 p.m., followed by the film "The Dambusters" and Annual Dinner. Branch Hut, R.A.F. Halton.

14th October

Cambridge.—Annual General Meeting followed by films:—"Song of the Clouds" and "Boeing 707." No. 1 Lecture Theatre, Cambridge University Engineering Laboratories. 8.15 p.m.

Halton.—Film Night. Branch Hut, R.A.F. Halton. 6.45 p.m.

17th October

Isle of Wight.—Aerial Photography. R. H. Smith. Clubhouse, Saunders-Roe Sports and Social Club, Church Path, E. Cowes. 6.30 p.m.

18th October

Birmingham.—The Control of Nuclear Power. L. W. J. Newman. At Hobsons Ltd., Wolverhampton. 7.30 p.m.

22nd October

Luton.—Junior Brains Trust. Napier Senior Staff Canteen, Luton Airport. 6.15 p.m.

28th October

Halton.—Aircraft in Agriculture. A/A Armiger, R.N.Z.A.F. (Junior Member). Branch Hut, R.A.F. Halton, 6.45 p.m.

DIARY—*Continued*

29th October
Boscombe Down.—A Review of the Future Prospects of Fleet Aviation. Captain D. R. F. Cambell. Lecture Hall, A. and A.E.E. 5.45 p.m.

30th October
Christchurch.—Aviation Fuels. Mr. Thompson. Kings Arms Hotel, Christchurch. 7.30 p.m.
Hatfield.—Titanium—A Broad Survey. P. L. Teed. de Havilland Restaurant, Hatfield. 6.15 p.m.
Luton.—Visit to Flowers Brewery.

4th November
Halton.—Film Night. Branch Hut, R.A.F. Halton. 6.45 p.m.

5th November
Luton.—Some Trends in the Development of Aircraft Electrical and Starting Systems. R. H. Woodall. Napier Senior Staff Canteen, Luton Airport. 6.15 p.m.

6th November
Reading and District.—Production Techniques. S. P. Woodley. Western Manufacturing (Reading), Upper Canteen. 6 p.m.

7th November
Isle of Wight.—Annual Dinner.

11th November
Halton.—Film Night. Branch Hut, R.A.F. Halton. 6.45 p.m.

12th November
Cambridge.—A Lecture of General Interest. Air Marshal Sir Victor Goddard. No. 1 Lecture Theatre, Cambridge University Engineering Laboratories. 8.15 p.m.

13th November
Brough.—Fourth Cayley Memorial Lecture. An Historical Survey of Naval Aviation. Rear Admiral Charles L. G. Evans, C.B.E., D.S.O., D.S.C. Royal Station Hotel, Hull. 7 p.m.
Chester.—Ultra-sonic Testing of High-Strength Aluminium Alloys. J. Crowther. Lecture Theatre, Grosvenor Museum, Chester. 7.30 p.m.
London Airport.—Brains Trust. Staff Restaurant, B.O.A.C., London Airport. 6.0 p.m.

15th November
Birmingham.—Ultra-sonic Testing of Light Alloys. J. Crowther. Eng. Centre, Birmingham. 7.30 p.m.
Brough.—Annual Dinner and Dance. Jackson's Ballroom, Hull.
Luton.—Branch Dinner. Leicester Arms, Luton.

18th November
Halton.—Film Night. Branch Hut, R.A.F. Halton. 6.45 p.m.

19th November
Luton.—Debate. Napier Senior Staff Canteen, Luton Airport. 6.15 p.m.

20th November
Christchurch.—Operation and Maintenance of High Altitude Aircraft. Personnel of the 7th Air Division, U.S. Army Air Force. Kings Arms Hotel, Christchurch. 7.30 p.m.
Halton.—Visit to U.S.A.F. Base.

27th November
Hatfield.—Experiences of an Airline Pilot. Captain O. P. Jones. de Havilland Restaurant, Hatfield. 6.15 p.m.

28th November
Halton.—Film Night. Branch Hut, R.A.F. Halton. 6.15 p.m.

Lighter-than-Air Re-union 1957

A Lighter-than-Air Re-union Dinner is to be held at Simpsons' in the Strand on Friday 22nd November 1957 at 7.00 p.m. Tickets will be 35s. per head and will include drinks before and during dinner.

The Dinner is open to all those who have flown in, or have been actively concerned in the design or construction of, lighter-than-air craft.

Those wishing to attend should apply to Mr. J. Fairley, Hon. Secretary, 7 Coopers Close, Kimpton, Herts.

Elections

The following is a list of new members and transfers of membership of the Society:—

Associate Fellows

Gordon Walter Bleasdale (*from Graduate*)
Jack Cohen (*from Graduate*)
Alan Robert Daffey (*from Graduate*)
Gilbert John Fairley
John Robert Denis Kenward (*from Graduate*)
David Ross Lane
Jagdish Chandra Pathak (*from Graduate*)
Henry Robert Warren (*from Graduate*)
Joseph Waters (*from Graduate*)
Frank James Wilkins
Kenneth William Wreghitt (*from Graduate*)

Associates

Allan McKenzie Cleveland
Harry McWhinnie Gartside
Angus James McDonald
Raymond Charles McNamara (*from Student*)
Richard Eric Pilkinton
Bruce William White (*from Student*)
Victor Robert Charles White
Winstone Harold Williamson (*from Student*)

Graduates

Leslie Quinn Hawe (*from Student*)
Derek Gordon Twist

Students

John Charles Fincher
Terence Nicholas Jones

Companion

Arthur Cecil Taylor

Acknowledgments

The Council wish to thank the following for the valuable additions to the Society's historical collections:—

J. D. Campbell, Esq., *Associate Fellow*, for a large collection of photographs, two official identification books of the First World World, and a copy of "Livre d'or de la Conquete de l'Air." The latter, published in a limited edition in 1909, has not hitherto been held in the Society's Library.

J. D. Joynt, Esq., of Manchester, for a copy of the First World War Air Board Technical Notes giving rigging instructions for twelve "vintage" aeroplanes. Mr. Joynt has also presented his personal lecture notes and "Record of Tests for Graduation" in connection with his training.

W. E. Larman, Esq., of Felpham, Bognor Regis, for a petrol tank cap stamped "H.M.A. Delta." This interesting relic of the 1912 airship was given to Mr. Larman by C.P.O. Hunt when they were together under Lieut. Maitland on the Airship C.P.14. Hunt lost his life in the R.101. The petrol tank cap has been given to the Society at the suggestion of Mr. H. F. Cowley, *Companion R.Ae.S.*

S. Mills, Esq., of Eastdene, Church Road, North Ferriby, for a copy of the Official Souvenir Programme of the Schneider Trophy 1929—the year in which Flying Officer Waghorn won the contest in the Supermarine S.6 at an average speed of 328·63 m.p.h.

H. C. Willis, *Companion*, of Simons Town, for two photographs of the 90 h.p. Curtiss flying-boat that was used to locate the German light cruiser "Königsberg" in November 1914. The pilot was Flight Sub-Lieutenant H. D. Cutler and his observer Midshipman A. N. Gallehawk. When making later reconnaissances, the Curtiss had to be force-landed and although the flying-boat was retrieved by a motor-boat in charge of Gallehawk, Cutler (who was flying alone on this occasion) was captured by the Germans. He remained a prisoner of war until November 1917 but the aircraft has an honoured place in Durban Museum.

The Journal of the Royal Aeronautical Society
WITH WHICH IS INCORPORATED THE INSTITUTION OF AERONAUTICAL ENGINEERS
VOLUME 61 OCTOBER 1957 NUMBER 562

ERNEST WALTER STEDMAN, *Fellow*—1888-1957

WITH the death of Air Vice-Marshal E. W. Stedman, C.B., O.B.E., Hon.F.C.A.I., F.I.A.S., F.R.Ae.S., in Ottawa on 27th March 1957, there passed from the scene one of the creators of the Royal Canadian Air Force, the officer responsible for its technical development, and a pioneer of Canadian aviation.

Air Vice-Marshal Stedman was a Canadian by adoption. Born at Malling, Kent, 21st July 1888, he took his secondary education at H.M. Dockyard School, Sheerness, and served two years' apprenticeship there before going to the Royal College of Science (A.R.C.S. First Class 1910). He was a Whitworth Scholar (1910) and studied civil and mechanical engineering at the City and Guilds Central Technical College, London, 1910-11.

After working as a draughtsman and engineer in industry, he was appointed in 1913 a Scientific Assistant in Aeronautical Research at the National Physical Laboratory and worked in the 4-foot wind tunnel there.

On the outbreak of war in 1914 he joined the Royal Naval Volunteer Reserve, transferring later to the Royal Naval Air Service, and being posted in 1918 to the newly formed Royal Air Force, in which at the end of the war, he held the rank of Lieutenant Colonel. At the time of the Armistice he was technical officer with a squadron of Handley Page heavy bombers awaiting favourable conditions for bombing Berlin.

On leaving the Service he joined Handley Page Limited as head of the technical staff and first crossed the Atlantic in 1919 as technical advisor for the Handley Page attempt to win the *Daily Mail* prize for the first non-stop Atlantic flight, abandoned when Alcock and Brown were successful.

Returning briefly to England, he again sailed for Canada in 1920 and resided in Ottawa until his death.

Colonel Stedman's arrival in Canada was opportune. The Air Board, newly established in 1919 for the control of aeronautics, was seeking a technical officer to undertake its engineering and scientific work. In October 1920 Colonel Stedman was appointed Director of Technical Services, and immediately began the organisation of the Board's technical branch.

On 1st January 1923 the Department of National Defence came into being and civil and military aeronautics were consolidated in the Canadian Air Force, which, shortly after, received the distinction " Royal," and became the Royal Canadian Air Force. The R.C.A.F. was first organised in three directorates and Colonel Stedman became Assistant Director, Supply and Research. His directorate comprised two sections—technical and research, responsible for the technical development of aeronautics, design and construction of aircraft, engines and accessories, maintenance, airworthiness and aero engineering generally; and equipment and supply responsible for supply of equipment, storekeeping, storage depots, and disposal of surplus.

During succeeding years, as the air force expanded and his responsibilities increased, he was successively Chief Aeronautical Engineer, Air Member for Engineering and Supply, Air Member for Aeronautical Engineering, Director General of Air Research and finally, in 1945, Air Member for Research and Development. He rose in rank in the R.C.A.F. from Wing Commander in 1924 to Air Vice-Marshal in 1941.

Air Vice-Marshal E. W. Stedman, C.B., O.B.E., Hon.F.C.A.I., F.I.A.S., F.R.Ae.S., R.C.A.F. (Retd.).

During his régime Air Vice-Marshal Stedman played a leading role in the technical development of Canadian aviation, from the dormant period following the 1914-18 War to the great era after the Second World War, with domestic and inter-continental air services in operation and a thriving industry.

The sound basis for technical organisation and development and for the intake and training of technical officers for which he was responsible, made possible the development of the R.C.A.F. from a service of 45 officers and 195 airmen at its inception in 1923, to the competent fighting force of 1939-45, the success of the British Commonwealth Air Training Plan, and the creation of a major industry in Canada.

Air Vice-Marshal Stedman was directly involved in most of the technical developments over this stirring period, including the adaptation of war surplus aircraft to civil uses, the surmounting of difficulties due to climate and terrain, the modification of imported aircraft to suit Canadian conditions, the design and construction of types of aircraft for Canadian requirements, such as forest patrol and fire suppression and aerial photography and survey, the practice for stressing for airworthiness, the development of aeronautical research and the establishment of a gas turbine industry.

Air Regulations 1920 required the certification of the airworthiness of aircraft. As no established procedure was available, Air Vice-Marshal Stedman prepared in detail a method for calculating the strength and stability of aircraft, which served until the familiar Handbook of Strength Calculations was issued by the Air Ministry.

Some of the earliest problems encountered in the use of aircraft designed elsewhere arose from the severity of the climate and included the design of skis and fitting them to wheeled aircraft, the starting and operation of engines at low temperatures, the effects of the wide temperature range on rigging and on structures of metal and wood, the

protection of crews in open cockpit machines, the hardening of rubber shock absorbers, and the shrinkage of wood.

During this period, the practice of aerial photography and survey was evolved and aircraft, camera mounts, plotting machines and techniques developed which placed Canada in the forefront in this new art and made possible the recent completion of the aerial mapping of the entire Dominion.

Air Vice-Marshal Stedman, recognising that the R.C.A.F. was essentially a technical service, was responsible for the decision to restrict the intake and training of pilots to graduates in applied science and the setting up, in 1923, of the plan to train selected undergraduates during successive summer vacations. He followed this up with the practice of sending officers, after some service experience, to the Imperial College of Science and Technology in London for post-graduate courses and of posting junior officers to the National Research Laboratories for service in the aerodynamics laboratory. A strong nucleus of trained aeronautical engineers was thus built up and he saw many of these officers reach senior rank in the R.C.A.F.

In 1920, an Associate Committee on Air Research was set up by the National Research Council at the request of the Air Board. Air Vice-Marshal Stedman served as a member of the committee from November 1920 until the committee lapsed in 1948, and was Chairman from 1936 to 1942.

He had, from the beginning, a keen interest in, and unusual appreciation of, the value of research in the development of aviation. As a member of the Associate Committee and in his successive appointments carrying increasing responsibilities for research, he strove constantly to expand the aeronautical research potential of Canada and throughout his career made effective use of the research facilities as they grew. At the end of the war, when appointed Air Member for Research and Development, he stated: "Research for defence is surely one of the most pressing needs of the post-war era."

When, in 1928, the National Research Council laboratories in Ottawa were projected, Air Vice-Marshal Stedman arranged with the President that a wind tunnel and water channel should be included, thereby initiating the development of the extensive aeronautical research facilities of the Council today.

Prior to the provision of the Ottawa laboratories, he supported and used the aerodynamics laboratory of the University of Toronto. When the laboratory was in difficulties in 1922, he arranged for a grant from the Department for the construction of a building to house the wind tunnel.

Air Vice-Marshal Stedman was also responsible for initiating the creation of an aeronautical museum recording the development of Canadian aviation. He arranged that, as aircraft engines became obsolete in the R.C.A.F., samples were reserved for museum purposes. In addition, he, personally, contributed many items including an ensign designed and approved for use at air stations of the Air Board, and a copy of his manual for airworthiness calculations.

Work in Canada on gas turbines was initiated by Stedman in 1942, leading to the construction and operation, under National Research Council auspices, of a cold weather test station at Winnipeg and, in July 1944, to the setting up of Turbo Research Limited, a Crown company for experimentation and scientific research and development of gas turbines. Air Vice-Marshal Stedman was a Director and one of those responsible for the decision that Turbo Research Limited should concentrate on the development of axial turbines, from which the Chinook, Orenda, and Iroqois have resulted. The creation of an aircraft engine industry, a goal toward which Air Vice-Marshal Stedman had constantly worked, thus came in sight.

Following his retirement from the R.C.A.F. in 1946, Air Vice-Marshal Stedman served briefly with the Defence Research Board as Scientific Advisor—Air. He witnessed the Bikini atom bomb tests in July 1946.

When Carleton College (now University) embarked on an engineering course in 1947, he joined the faculty as Assistant Professor and Chairman of the Department and served there until 1954.

Latterly he had been engaged on the writing of his memoirs, which, unfortunately, he was unable to complete.

Air Vice-Marshal Stedman was a Fellow of the Royal Aeronautical Society and was awarded the Silver Medal of the Society in 1922. He was a Fellow of the Institute of the Aeronautical Sciences and an Honorary Fellow of the Canadian Aeronautical Institute. He was a Member of the Council of the Institution of Civil Engineers and was awarded the Bayliss Prize of the Institution in 1910. A member of the Engineering Institute of Canada, he had served as a Chairman of the Ottawa Branch and member of Council.

Air Vice-Marshal Stedman was awarded the O.B.E. for his services in the First World War, the C.B. in 1944, and was a Commander of the U.S. Legion of Merit.

He was the author of numerous technical papers on aeronautical and engineering subjects.

A keen stamp collector, he possessed a notable collection. His recreations included ski-ing, skating, swimming, canoeing, and motoring.

Reserved, rather shy, sincere, Air Vice-Marshal Stedman was a staunch, but undemonstrative, friend. During the flight of HMA-R100 to Canada in August 1930 he was a passenger on the Canadian flight and also on the return flight to England. The loss of many friends in the R101 disaster shortly after was a great shock to him.

He was an engineer of wide interests and an inquiring mind, constantly extending his knowledge. During undergraduate days, he enrolled for additional courses outside the regular curriculum and, later, obtained leave, in 1931, to take a special course in metallurgy at McGill University.

Well-known in aircraft plants and government offices at home and abroad as a result of his tours to observe developments, his views and suggestions based on his wide experience in aeronautics were everywhere welcomed and respected. For example, the introduction of the seat-pack type parachute into the R.A.F. resulted from a discussion in the office of Sir Hugh Trenchard, and information acquired as a result of his unsatisfactory experience with benzol and alcohol in gasoline, on the newly discovered tetra-ethyl lead as an anti-knock agent, proved helpful to the Royal Aircraft Establishment in assessing the merit of TEL.

Air Vice-Marshal Stedman's foresight, sound judgment and aeronautical knowledge during the critical formative years provided the foundation on which the R.C.A.F., as a technical service, developed to its present stature.

Canada owes him a debt of gratitude for his great contributions to the advancement of the Royal Canadian Air Force and to Canadian aviation.

His associates and many friends throughout the world of aeronautics will have learned of his passing with sorrow.—J. H. PARKIN.

Materials for Aircraft Structures Subjected to Kinetic Heating*

by

A. J. MURPHY, M.Sc., F.I.M., F.R.Ae S.

(*Principal, College of Aeronautics*)

1. Introduction

The requirements of aeronautical engineers have been one of the greatest stimulants of metallurgical research and development during the present generation. Materials for the piston engine and the airframe had shared about equally the attention of the research metallurgist from the early twenties to about 1945. Then the gas turbine appropriated the major effort in metallurgical research because it was clear that progress with this propulsive unit was inescapably dependent upon the production of metals and alloys capable of giving reliable service at ever increasing temperatures. Here it was a question of evolving new materials having characteristics for which no appreciable demand existed before the advent of the gas turbine. The past five years have seen the airframe come back into the picture metallurgically, because of the new concern with fatigue and because supersonic speeds are bringing with them service temperatures for the airframe higher than anything arising while speeds of flight were comfortably subsonic. The considerations of materials for the hotter components of gas turbines on the one hand and of kinetically heated airframes on the other have this important distinction, that while with the gas turbine it has been, as has been said, mainly a question of developing new materials, with the airframe what is involved is essentially making a judicious selection from materials which are already known, and mostly in current production. It is with this latter task that this paper is concerned.

To define the terms and area of the problem, the temperatures which have to be reckoned with must first be known. We must then consider which mechanical and physical properties of the structural material are of significance to the efficient service of the airframe. It is necessary to ascertain the relevant characteristics of available materials. Finally, to establish a secure basis for successful application of present materials and possible modifications to improve them, rational explanations of their performance should be sought in the light of metallurgical science.

Given the external conditions of relative air speed and laminar or turbulent flow, the characteristics of the material influencing the temperature attained generally and locally in the structure are the emissivity, thermal conductivity, and specific heat. Those directly affecting the strain, and in the case of restricted dilatation the stress, in the material are the coefficient of thermal expansion and the elastic moduli. Also involved in the distribution of strain is the plastic yielding of the material by creep and perhaps its internal damping capacity. Finally there remains the basic requirement of mechanical strength, as measured by tensile and compressive strength, proof stress and fatigue strength, with amenability to working and freedom from brittleness. Corrosion resistance must be adequate initially and must not be unduly impaired during service.

Attention here is directed principally to speeds up to a Mach number of 4. This is a largely arbitrary choice but it is perhaps acceptable for the present discussion because it involves a range of temperatures in which the selection of structural materials presents greater scope than would more extreme conditions.

2. Temperatures Attained

Although the easily remembered rule is appreciated that the air temperature, $T°C$ at the outer limit of the boundary layer is given by $(V/100)^2$, where V is speed in miles per hour, the maximum temperature of the skin of the aircraft is lower than this. The term "saturation temperature" has been introduced to describe the steady temperature which would eventually be reached if the operating conditions remained constant for long enough. For the guidance of metallurgists Hartshorn[1] gives a number of examples of saturation temperatures attained at various values of M, height, and emissivity, for turbulent or laminar conditions. Fig. 1 shows actual saturation temperatures to be expected in bodies of length 10 to 100 ft. moving in turbulent air at a number of speeds up to a Mach number of 10, at different heights and with various emissivities. Increase in height, emissivity, length of vehicle or laminar flow reduces the temperature attained. Examples worked out by Hartshorn for specific flight histories suggest that the saturation temperatures in Table I may be assumed at heights of 50,000 to 75,000 ft., with emissivity of 0·9, for the purpose of evaluating possible materials of construction.

The metallurgist is particularly grateful to Hartshorn for putting forward these figures since the literature is at present in a most confused state about the temperatures in actual structures likely to be produced by

*A Section Lecture given on 11th December 1956.

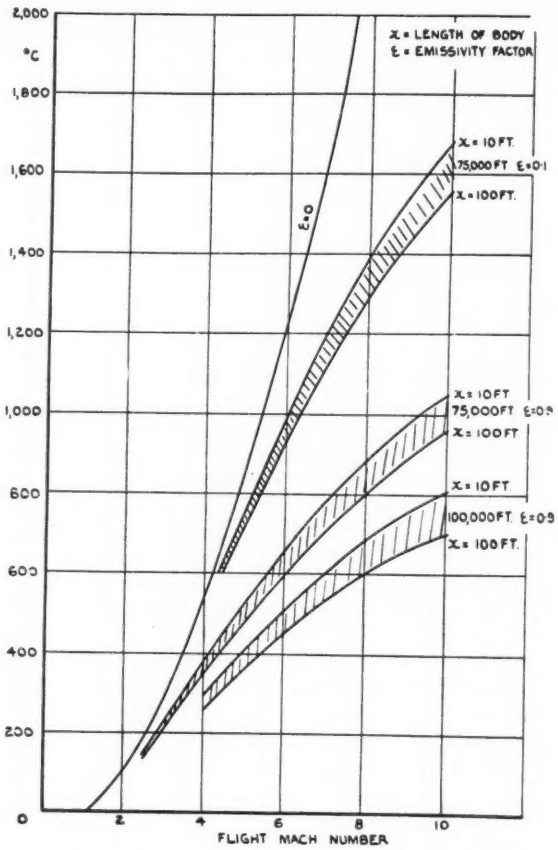


FIGURE 1. Saturation temperatures for various operating conditions.

kinetic heating. In general, estimates of the temperatures are higher than those just quoted. Thus recent papers and discussions have contained references such as: "for speeds of Mach 2 and Mach 3 skin temperatures would be of the order of 200-300°C with turbulent air flow"; "at Mach 3 the estimated temperature due to laminar flow only would be 80°C, but that due to turbulent flow would be 225°C"; "skin temperature in Lockheed F104 Starfighter at 1,320 mp.h. was 177°C". Meikle[2] deduced from Hartshorn's data the values given in Table II for the temperatures reached at 75,000 ft. with an emissivity factor of 0·9 and ambient temperature −56·6°C.

In some instances it is clear that writers are not distinguishing between temperatures in the air and temperatures in the metal, but in others there appears to be a genuine conflict of evidence or, or opinion. As will be shown later, precise knowledge of the temperatures to be reckoned with, and of the times in which they are attained, is essential if efficient use is to be made of available materials of construction.

TABLE I

Mach number	*Saturation temperature °C. (ambient temperature −56°C.)*
2·0	100
2·5	150
3·0	200
3·5	300
4·0	370

3. Materials Available

The materials to be discussed in relation to their suitability for structures operating within this envelope of temperatures are: aluminium alloys, magnesium alloys, titanium alloys, steels and nickel alloys.

3.1. ALUMINIUM ALLOYS

Aluminium-base alloys have proved to be extremely satisfactory as the structural material of aircraft operating under the conditions which have been regarded up to the present as normal. In respect of speed of flight this means, say, 0·7*M* and cannot involve kinetic heating of more than about 25°C as a maximum. The modern alloys are typified by L.65 (D.T.D.364) and the American 2024S-T4 in the aluminium-copper-magnesium family and D.T.D.687 (American 7075S-T6) in the aluminium-zinc-magnesium category. For service at normal temperatures these offer the aircraft designer and constructor the advantages of high strength-weight ratio, ease of fabrication, corrosion-resistance and relative cheapness. Their modulus of elasticity, 10,000,000 lb./in.^{2}, is only one-third of that of steel but this is largely offset by the density which is also about one-third of that of steel. Compared with steel all aluminium alloys lose strength more rapidly as the temperature rises above 150°C and somewhere between 100 and 300°C there is a temperature above which aluminium alloys cannot offer sufficient strength or stability to permit their use in an aircraft structure.

The investment of capital and skill in the aluminium industry and of scientific effort in research on light metals has been so great that it is of the highest importance to ensure that the technical and economic exploitation of the metal is continued for as long as possible against the increasingly unfavourable odds. To this end it is necessary to examine closely the metallurgical basis for the mechanical properties of the aluminium alloys with which there is so much experience in airframes. In any event the more precisely the spectrum of temperatures which a structure will encounter can be known and the time for which it will be exposed to the various ranges of temperature, the more effectively can the fullest use of the potential advantages of aluminium alloys be used. As will be seen later, an over-estimate of the maximum temperature to the extent of 50°C would be sufficient in some circumstances to involve the constructor in extravagant

TABLE II

Mach number	*Temperature °C*
2·0	100
3·0	200
4·0	400
5·0	500
6·0	600

use of costly materials and difficult processes of fabrication.

The strong aluminium alloys on which present techniques of airframe construction have been developed owe their mechanical properties to heat treatment; essentially the heat treatment comprises a two-stage process. The first stage is the "solution treatment" in which the material is heated to a temperature of 450 to 530°C to take a good proportion of the alloying additions into solid solution in the aluminium matrix; they are retained in forced solution by rapid cooling, generally through quenching in water. In this condition the material is soft. The point to be emphasised is that, as regards its inner structure, the "solution treated" material is in an unstable state: its tendency is always to move towards an equilibrium condition by allowing some of the dissolved metals to be precipitated as particles of compound constituents. In some alloys this reversion towards equilibrium occurs spontaneously at the ordinary temperature, in others some heat has to be applied and maintained for precipitation to proceed. According to whether precipitation has occurred spontaneously or has been brought about by heating, the second stage of "heat treatment" is described as "natural ageing" or "artificial ageing". It is in this stage that the increases in strength and hardness occur. The temperature at which age-hardening takes place after solution treatment in aluminium alloys may be said to be between room temperature, say 15°C and about 200°C. It will be observed that this corresponds approximately with the range of saturation temperatures resulting from kinetic heating in structures at flight speeds up to a Mach number of 3. Evidently aluminium alloys going into service as structural members of aircraft flying at speeds in excess of a Mach number of 2 will be subjected to an "artificial ageing" additional to that which they may have received initially.

We must distinguish here between the permanent effects of the reheating during service on the properties as measured when the component has cooled down again to the ordinary temperature (the "recovery values") and the changes in strength and hardness apparent when the measurements are made on the article at the elevated temperature of service.

D.T.D.683 is an example of the aluminium-magnesium-zinc family of structural alloys. Its normal heat treatment process is: solution treatment, 465°C, precipitation 16 hours at 130°C. In this condition it is the strongest structural aluminium alloy at the ordinary temperature in regular use today. It is, however, more sensitive than the aluminium-copper-magnesium type to rise of temperature. At 120°C it has already lost its superiority over, for instance, L.65 in respect of both strength at elevated temperature and recovery values on return to the ordinary temperature. Except for some special applications in which the component is raised suddenly to the high temperature, and dwells at that level for not more than a few minutes, the aluminium-magnesium-zinc alloys (D.T.D.683, D.T.D.687, D.T.D. 363, 7075S) do not rank for consideration in structures heated kinetically, or otherwise, above about 100°C.

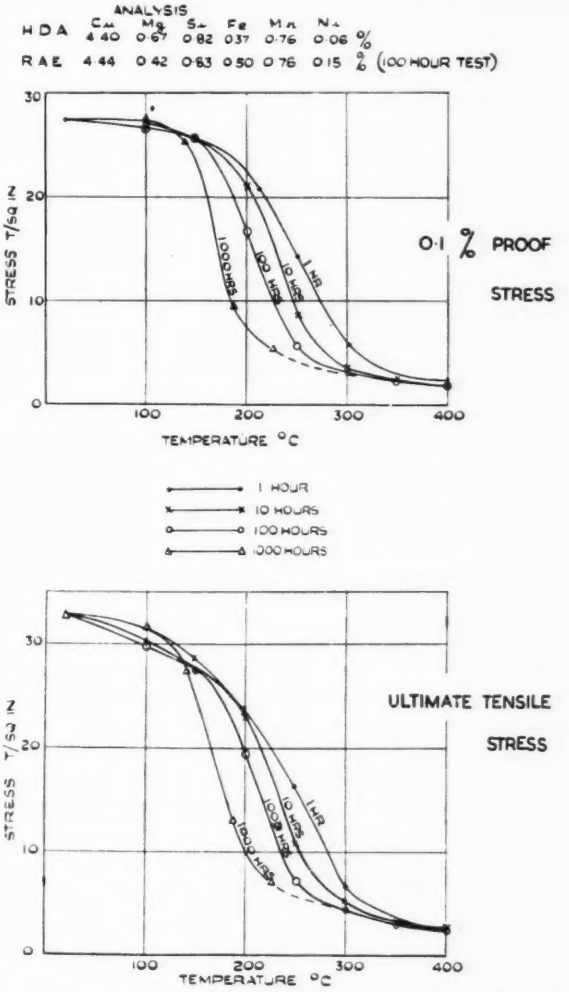


FIGURE 2. 0·1 per cent proof stress and ultimate tensile stress of D.T.D. 364 after heating for 1, 10, 100 and 1,000 hours and tested at the elevated temperature. (Ref. 2).

Consider now the aluminium-copper-magnesium alloys. This is the family of the original Duralumin and L.65 (D.T.D.364) may be taken as a representative of the artificially aged members. The standard heat treatment is: solution treatment, 510°C, precipitation, 16 hours at 175°C. The D.T.D. Specification calls for the minimum values: 0·1 per cent proof stress, 24 tons (53,800 lb.) in.^{2}, ultimate tensile stress 28 tons (62,700 lb.) in.^{2}, elongation 8 per cent in light extrusions.

Figure 2[2] shows what happens to the proof stress and tensile strength of this fully heat-treated material when it is held at different temperatures from 15 to 400°C for periods varying from one hour to 1,000 hours. The properties are measured at the elevated temperature.

It will be seen that the re-heating has not brought the tensile strength below the minimum of the specification until the temperature has been raised to about 170°C for one hour's heating and to 140°C for an

exposure of 1,000 hours. If a deterioration in tensile strength of 20 per cent below the specified value could be tolerated, the permissible temperature would be: for one hour's exposure, 210°C; for 10 hours, 200°C; for 100 hours, 180°C; for 1,000 hours, 160°C.

These curves relate to tests in which the specimens were held unstressed at the temperatures shown. For the longer exposures it is necessary to consider what happens when a load is maintained throughout the period. In this case creep occurs and, as shown in Fig. 3, in 1,000 hours at 150°C, 0·1 per cent strain is produced by a stress of 12 tons in.^{2} continuously applied, whereas to cause the same strain in a specimen not stressed until the termination of this period a stress of 23 tons in.^{2} was required. It may be that 1,000 hours is an unduly long time as a basis for evaluating materials for service during which the total time of exposure to pronounced kinetic heating may only amount to a small proportion of the operating life of the aircraft. Perhaps 100 hours would be a more realistic basis for many types of vehicle and service.

In this connection a word may be said to caution against attributing to creep tests too high a degree of consistency and reproducibility. The actual strain in a specimen undergoing creep is highly sensitive to factors such as the history of working and annealing before loading, and the grain size. Battelle report[3], for instance, that in creep tests of 18S.T6 (similar to L.65) rod at 400°F (204°C) under a stress of 15,000 lb. in.^{2} a total deformation of 0·5 per cent was recorded in 60 hours in a fine-grained specimen, in 300 hours in a medium grain size and in 600 hours in a coarse-grained specimen. These correspond to minimum creep rates of 120, 40 and 15 millionths per hour.

Since in an aluminium alloy, such as L.65, reheating above 150°C causes a resumption of the precipitation process, it may be profitable to consider service of this material in a kinetically heated aeroplane as a continuation of the second stage of the heat treatment by which its original mechanical properties were conferred. Fig. 4* shows the progressive change in tensile strength in 26.S alloy (a current version of L.65) sheet when it is maintained at a number of different temperatures from 15° to 195°C immediately after quenching in the solution treatment.

The significant features in these curves to which attention is directed are the following:

(*a*) As quenched from the solution temperature the material is in its softest condition:
the ultimate tensile stress is
48,000 lb. in.^{2} (21·5 t.s.i.)
with 0·1 per cent proof stress
21,000 lb. in.^{2} (9·5 t.s.i.)

(*b*) On resting at *room temperature* the material hardens spontaneously and continuously, attaining in 32 hours a proof stress 39,000 lb. in.^{2} (17·5 t.s.i.).
The age-hardening is still proceeding after 300 hours.

*Data from Aluminium Laboratories Ltd.

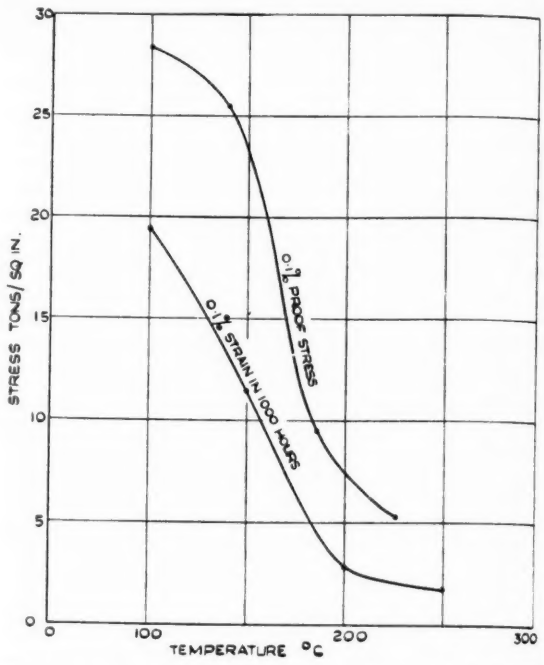


FIGURE 3. Stress to produce 0·1 per cent creep strain in 1,000 hours compared with 0·1 per cent proof stress after heating for 1,000 hours D.T.D. 364.

(*c*) At *150°C* the 0·1 per cent proof stress rises continuously attaining 54,000 lb. in.^{2} in 20 hours.

(*d*) At *160°C* the 0·1 per cent proof stress is over 54,000 lb. in.^{2} in 5½ hours, passes through a maximum at 63,000 lb. in.^{2} in 24 hours, falling thereafter to 61,000 lb. in.^{2} in 36 hours, when it is still decreasing.

(*e*) At *170°C* the proof stress is 54,000 lb. in.^{2} in 2½ hours, passes through a maximum at 62,000 lb. in.^{2} in 14 hours, falling thereafter to 60,000 lb. in.^{2} in 36 hours.

(*f*) At *180°C* the proof stress is 54,000 lb. in.^{2} in 2 hours, and is a maximum at 62,000 lb. in.^{2} in 5 hours, falling to 54,000 lb. in.^{2} in 48 hours.

(*g*) At *195°C*. The maximum, at 59,000 lb. in.^{2}, is already attained in 2 hours, and the proof stress has fallen to 52,500 lb. in.^{2} in 36 hours.

The most important conclusions from the results are:

1. At 150°C the originally soft material attains the 0·1 per cent proof specified as a minimum in L.65 (24 tons per sq. in., 54,000 lb. in.^{2}) in 20 hours and shows no signs of softening during the course of the observations, 52 hours.

2. At 160°C there is the first indication of the age hardening passing through a maximum.

3. Even at 195°C, at which proof stress rises to the specification minimum of L.65 within one hour, after heating prolonged for 36 hours the proof stress of the

alloy is only just falling below the specification requirement.

The normal period of precipitation treatment is 12 to 16 hours at a temperature of 155 to 185°C. It can be seen that these conditions coincide fairly closely with the maxima on the ageing curves—which is, of course, the reason for selecting them. There is at present a preference among metallurgists to continue the precipitation treatment beyond the hump on the curve, rather than to stop before the maximum strength has been attained. The main justification for thus "over-ageing" rather than "under-ageing" is that under-ageing is liable to leave undesirable internal stresses in the material. This appears to have been demonstrated in D.T.D.683, but it is doubtful whether there is any corresponding effect, to a significant degree, in L.65 and similar alloys.

There would seem to be a case for examining more closely this question of "under" or "over" ageing in aluminium alloys for kinetically heated structures. Suppose the critical service condition involves a temperature of 180°C in the structure and that the structural material would be satisfactory so long as it had a proof stress not less than 57,000 lb. in.$^{-2}$. If it were made of material having the ageing characteristics shown in the curves for alloy 26S, and the material had been heat treated for 16 hours at 180°C, the proof stress on going into service would be 61,000 lb. in.$^{-2}$ After 17 hours of service at 180°C the material would have over-aged to such an extent that the proof stress would have fallen to the minimum acceptable figure of 57,000 lb. in.$^{-2}$. On the other hand, the graphs show that two hours precipitation treatment would have been sufficient to give the required proof stress. If the material had been put into service (again at 180°C) in this condition (*i.e.* after two hours ageing) a service life of 31 hours would have been possible—a gain of 14 hours over what would have resulted from the conventional heat treatment. This example is purely hypothetical—one would not reckon on as high a degree of reproducibility of ageing characteristics—but for types of aircraft in which only a limited period of exposure to high temperatures was to be anticipated, advantages might be taken of considerations such as the one outlined.

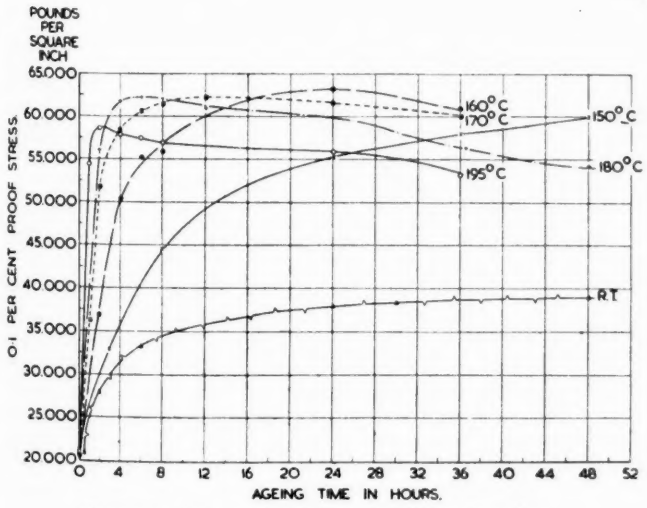


FIGURE 4. Ageing of A.C. 26S. aluminium alloy sheet.

There is reason to believe that the fatigue of strong light alloys is basically a process of local over-ageing, and consequent softening, caused by the alternating strains. It would not be surprising if endurance in fatigue were improved by under-ageing. In creep also it is known that the condition in respect of ageing can have an important influence.

Considerations of superior fatigue performance lead sometimes to a preference for naturally aged Duralumin (*e.g.* to Specification D.T.D.610) in place of the artificially aged version (D.T.D.546). It is possible to deduce from the data just discussed what the condition of such material would be in parts of the skin which had experienced kinetic heating within the ranges of temperature and time discussed. Incidentally, for the temperatures considered, *i.e.* up to 200°C, the effects of successive exposures to kinetic heating to the same temperature may safely be regarded as additive. This would no longer be true when the reheating temperature exceeded about 320°C, because then a certain amount of precipitate would be re-dissolved, with the possibility of subsequent re-precipitation at lower temperatures and renewed age-hardening.

Deterioration in the sense of progressive loss of strength under kinetic heating can be kept under observation by hardness tests made in the areas where the highest temperatures are experienced. The situation in this respect is better than when fatigue is the cause of deterioration: the damage due to fatigue may remain for a long time so localised that it is impossible to know where to apply any test to reveal a significant change in properties.

The microstructural changes which are associated with the continued ageing of heat-treated aluminium alloys on kinetic heating are liable to affect the behaviour under corrosive conditions. The effect is only serious for Duralumin alloys which are put into service in the naturally aged condition and are then exposed to a temperature in the region of 100°C for some hours. This is due to precipitation at grain boundaries rendering the material subject to inter-crystalline corrosion. Duralumin clad with pure aluminium does not suffer this impairment of its corrosion resistance so long as the cladding is not penetrated.

In the development of the conventional aluminium alloys it was found that an improvement in the mechanical properties in the temperature range 200° to 350°C could be achieved by making modifications to the composition. The American 24S alloy has a higher magnesium content (1·5 per cent compared with 0·5 per cent) than the standard Duralumins and this affects its age-hardening characteristics in such a way that the strength at 200° to 250°C is improved. Y-alloy, an early British piston alloy, has 1·5 per cent magnesium

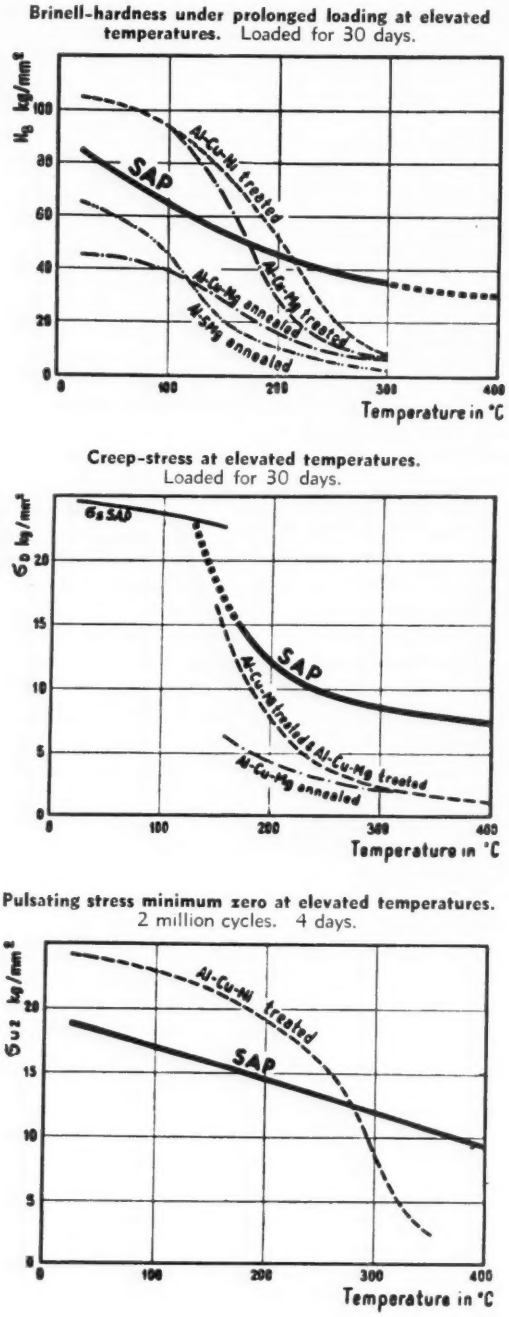


FIGURE 5. As S.A.P. is not changed by heating, the duration of the test has no influence on the results. At temperatures higher than 250°C. the fatigue properties of S.A.P. surpass those of any aluminium alloy which under such conditions rapidly becomes annealed. Under proplonged loading S.A.P. surpasses the values of heat treated alloys at 200°C. (Ref. 4).

and 2 per cent nickel; R.R.58 and R.R.59 are of similar type. The most interesting recent development in this direction is the alloy R.R.57 which contains no magnesium or nickel. Essentially it is a simple alloy of aluminium, with 6 per cent copper, 0·25 per cent manganese and 0·1 per cent titanium. Probably its good qualities at elevated temperature are connected with the absence of low-melting point constituents. The temperatures of solution treatment (530-535°C) and ageing (215°C) are higher than with other strong aluminium alloys. The 0·1 per cent proof stress of R.R.57, which is 15 tons per sq. in. at the ordinary temperature, falls to about 7·5 tons per sq. in. in a short-time test at 300°C; the valuable feature is that on maintaining it at this temperature for 200 hours there is only a very slight further decrease. This indicates a good measure of structural stability at this temperature. R.R.57 is applied mainly in forged pistons and compressor blades. If it can be produced in the form of light extrusions and rolled sheet it should have interesting possibilities of utilisation in kinetically heated structures.

An entirely new departure from the conventional aluminium alloys is "Sintered Aluminium Powder" (S.A.P.), a powder metallurgy product, made by compacting flakes of aluminium coated with aluminium oxide, the proportion of oxide being generally 10 per cent in rolled sheet and 13 per cent in extrusions. The hardness and strength of S.A.P. depend on permanent dispersion hardening as distinct from the precipitation hardening which is a common feature of all other strong aluminium alloys. Whereas at elevated temperatures the age-hardened materials change more or less rapidly towards an equilibrium condition, eventually leading to loss of strength, S.A.P. remains structurally completely stable at all temperatures short of the melting point of aluminium (660°C). Its strength and hardness fall off as the temperature rises, but less rapidly than in Duralumin. S.A.P. recovers its original properties completely on cooling to room temperature, even after as long as 1,000 hours soaking at 550°C.

Typical mechanical properties of S.A.P. in sheet form are:

0·1 per cent proof stress	15 tons in.^{2}
Ultimate tensile stress	24 tons in.^{2}
Elongation	8 per cent
Brinell hardness number	100

The proof stress and tensile strength are very similar to the minimum values of naturally aged Duralumin, although the properties normally obtained in the latter are appreciably higher.

The three diagrams of Fig. 5[4] illustrate the superiority of S.A.P. in respect of prolonged tests over all conventional aluminium alloys at temperatures above 250°C.

Because there are no elements in solid solution in the aluminium matrix, the electrical and thermal conductivities of S.A.P. are high: in extrusions containing 13 per cent Al_2O_3 they amount to 80 per cent of the values for 99·5 per cent pure aluminium. This is much greater than in the usual strong heat-treated alloys, in which the conductivities are little more than one-half of those in the pure metal. Young's modulus is 11·3 million lb. in.^{2} in extrusions and 10·7 million in sheet.

The cold forming properties of S.A.P. sheet are not

particularly good, but on hot forming at 400° to 450°C it can be bent over a radius of 2·5 × thickness.

It is natural to speculate as to the benefit which might be gained by applying the techniques of S.A.P. to other metals. A first requirement seems to be that the oxide must not be soluble in the solid basis metal at any temperature. This rules out titanium, in which oxygen is readily soluble. The method might work with magnesium and nickel. Where the oxide is soluble another compound may be used, for instance, a carbide or a sulphide. This may open the way to important advances in the development of metallic products for service at elevated temperatures.

The discussion so far has been concerned with aluminium alloys, the limiting upper temperatures at which their mechanical properties fall away seriously and the metallurgical reasons for these limitations. So great a wealth of experience in the use of aluminium alloys has been accumulated that an element of intuition has become added to strict engineering analysis in the application of the material to aircraft structures operating at normal temperatures. At elevated temperatures certain characteristics of structural materials of no importance at ordinary temperatures become of significance. In this connection the thermal conductivity (or, more strictly, thermal diffusivity) and the coefficient of thermal expansion are obviously relevant in affecting the strain under non-uniform heating, while the modulus of elasticity at the temperature has to be known to permit the calculation of the associated stress.

4. Structural Efficiency at Ordinary Temperatures

Assuming the tensile strength, proof stresses in tension and compression and the elastic moduli at the operating temperature are known, the considerations involved in structural efficiency, so far as selection of materials is concerned, are the same as at ordinary temperatures with certain additional limitations when creep strain is anticipated.

Materials which come into consideration for service at temperatures up to 400°C are shown in Table III in order of increasing density.

At the ordinary temperature the order of merit for structural efficiency depends on the criterion which is adopted. If the basis of selection is simply resistance to direct stress, the ranking is according to the quotient of tensile strength (or proof stress in tension) divided by specific gravity. This puts the two titanium alloys easily at the head of the list (U.T.S./d=15 to 17, where U.T.S. is in tons/in.^{2} and the density d is in gm./cc.) the aluminium alloys (without S.A.P.) and the two steels in a group (10 to 11), then Nimonic 90(8), and finally S.A.P. and the magnesium alloy (7) at the bottom. Where the case considered is resistance to buckling in compression the quantity to be reckoned with is the square- (or cube-) root of the modulus of elasticity divided by the density, *i.e.* $E^{\frac{1}{2}}/d$ or $E^{\frac{1}{3}}/d$.

This puts the magnesium alloy at the top of the list ($E^{\frac{1}{2}}/d$=1·4, where E is in million lb./in.^{2} and d is in gm./cc.), the aluminium alloys, including S.A.P., next (1·1), then the titanium alloys (0·9) and, at the bottom, the steels and Nimonic 90 (0·7). This is the order for all buckling criteria in which Young's modulus is to a lower power than the density. It will be seen later how these orders change at higher temperatures.

5. Thermal Stresses

Materials in kinetically heated structures have to withstand not only the stresses due to flight, but also the thermal stresses. The physical properties of the

TABLE III

STRUCTURAL MATERIALS

Magnesium alloy	Zn %	Zr %	Th %	Mg remainder
ZTX	1·0	0·7	2·75	

Aluminium alloys	Cu %	Mg %	Si %	Fe %	Mn %	Ni %	Zn %	Cr %	Ti %	Al %
L65, DTD364, 546, 603, 610, 646	4·2	0·5	0·8	—	0·8	—	—	—	—	remainder
DTD363	1·8	2·1	—	—	0·2	—	6·5	0·1	—	remainder
DTD683, 687, RR77	0·4	2·7	—	—	0·5	—	5·3	—	—	remainder
75S	1·5	2·5	—	—	—	—	5·5	0·2	—	remainder
RR57	6·0	—	—	—	0·25	—	—	—	0·1	remainder
RR58	2·2	1·5	—	1·0	—	1·2	—	—	0·1	remainder

Steels	C	Mn	Ni	Cr	Mo	Cu	Nb	Al	Fe
Rex 448	0·1	1·0	0·3	11	0·8	—	0·5 (V0·15)	—	remainder
Armco 17-7PH	0·07	0·7	7	17	—	—	—	1	remainder
FV520	0·07	—	5	16	2	2	—	—	remainder

Nickel alloys	Fe	Cr	Co	Ti	Nb	Al	Ni
Nimonic 90	<5	20	17·5	2·5	—	1·2	remainder
Inconel X	7	14	—	2·5	1	0·8	remainder

Titanium alloys	Al	V	Sn	Ti
6Al4V	6	4	—	remainder
I.C.I. 371	2·75	—	13	remainder

TABLE IV

Material	*Thermal stress modulus* $E\alpha$ *lb. in.² deg. C.*
Titanium alloys	157
Magnesium alloys	173
Aluminium alloys	243
Alloy steels	365
Nimonics	421

material which principally influence the thermal stresses are: thermal conductivity K, coefficient of thermal expansion α, specific heat h, modulus of elasticity E, and density d. Density comes into the picture because the thermal diffusivity (K/hd) decides the quantitative capacity of the material to transfer heat by conduction.

In transient heating when a severe temperature-gradient exists between, say, the outer skin and a supporting web of the same material, the stress in the web is proportional to the temperature difference T, the modulus of elasticity E and the coefficient of thermal expansion α. The term "thermal stress modulus" has been applied to the quantity $E\alpha$. For some of the materials already considered for kinetically heated structures values of $E\alpha$ (lb. in.², deg. C) taking values of E and α at temperatures around 15°C are given in Table IV.

The more appropriate values of E and α, however, would be those for an intermediate temperature between the lowest initial, and the highest final, temperatures. If, as an example, values at 200°C are taken the list is as given in Table V.

Thermal stresses are minimised by a low modulus of elasticity, but for rigidity and stability of the structure a high value of the modulus is desired. The dilemma is not so troublesome as might be expected because among these materials, at least at the ordinary temperatures, there is an almost uniform proportionality between E and density. For each of the five materials which have been discussed the ratio (E/d) in lb./inch units is close to 100×10^6 at the ordinary temperature. Up to 200°C the moduli of the steels and nickel-base alloys have fallen by only 4 to 5 per cent, and the

TABLE V

Material	*E at 15°C. million lb. in.²*	*E at 200°C. million lb. in.²*	*α 150°-200°C.*	*Thermal stress modulus* $E\alpha$ *at 200°C. lb. in.² deg. C.*
Titanium alloys	17·3	16·3	91	148
Magnesium alloys	6·4	5·8	27	157
Aluminium alloys	10	9	24·3	219
Alloy steels	29·4	28·2	12·4	349
Nimonics	32·4	31	13	403

TABLE VI

Material	*Thermal conductivity*	*Specific heat*	*Density*	*Thermal diffusivity*
Titanium alloys	0·04	0·13	4·5	0·07
Magnesium alloys	0·3	0·245	1·8	0·68
Aluminium alloys	0·35	0·23	2·7	0·56
Alloy steels	0·06	0·11	7·8	0·07
Nimonics	0·03	0·11	8·3	0·03

titanium alloys by 6 to 7 per cent, as is seen in Fig. 6[5]. In the magnesium and aluminium alloys for the same temperature range E has fallen by 9 to 10 per cent. The accompanying changes in α, the coefficient of thermal expansion, are of the opposite sign, *i.e.* the coefficients increase with rise of temperature but the percentage increase is only in the order of $3\frac{1}{2}$ to $5\frac{1}{2}$ per cent, with the lighter metals least affected.

Thermal stresses, other things being equal, will be directly proportional to the modulus of elasticity, whereas when rigidity is the dominant consideration in design it is only a fractional power (perhaps the cube root) of the modulus which comes into the account. A low modulus therefore will give a bigger gain on the thermal stress account than the corresponding loss in rigidity.

Thermal stresses would be expected to be less the higher the thermal conductivity. The quantity involved is really the thermal diffusivity, *i.e.* the thermal conductivity divided by the product of the specific heat and the density. The values are given in Table VI in c.g.s. units.

It is seen that in their ability to transfer heat by conduction along a given section the alloys of magnesium and aluminium are almost ten times as effective as titanium alloys and stainless steels, and that these in turn are twice as effective as the nickel base alloys.

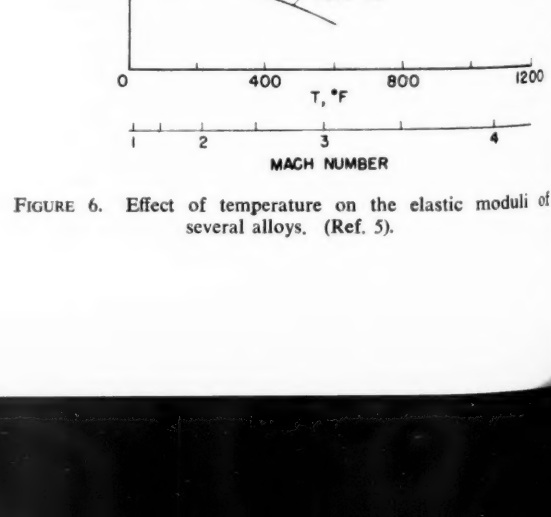


FIGURE 6. Effect of temperature on the elastic moduli of several alloys. (Ref. 5).

TABLE VII

Material	Thermal stress modulus / Thermal diffusivity c.g.s. units × 10^3
Titanium alloys	147
Magnesium alloys	16
Aluminium alloys	27
Alloy steels	350
Nimonics	860

For a given heat input the resulting thermal stress would seem to be proportional to:

$$\frac{\text{thermal stress modulus}}{\text{thermal diffusivity}} \quad \text{or:}$$

$$\frac{E\alpha.h.d.}{K}$$

where h = specific heat, d = density, K = thermal conductivity

This coefficient or modulus for the five materials is shown in Table VII.

From a consideration of the physical properties of these five structural materials it can be seen that there is a *prima facie* case for magnesium and aluminium alloys as the least likely to give rise to high thermal stresses. Titanium comes next, then the steels and finally the nickel alloys. The relative orders of magnitude are: magnesium and aluminium unity, titanium 7, steel 16, nickel alloys 40.

6. Alloys of Low Thermal Expansion

This may be an appropriate stage at which to call attention to a class of material which I have not seen mentioned previously in discussions of kinetically heated structures. I have in mind alloys based on the low-expansion iron-nickel alloy known as Invar.

Invar is an alloy of 64 per cent iron and 36 per cent nickel. Over the range of temperature 15° to 200°C its coefficient of thermal expansion is 4×10^{-6}, the minimum value being $1{\cdot}0 \times 10^{-6}$ at 0°C. The latter is about 1/10th of that of steel, 1/20th of that of aluminium. The anomalous low expansion at this composition is probably the resultant of two factors, one the normal dilatation characteristic of nickel-base and iron-base alloys, of the order of 12×10^{-6} per °C and the other an *increase* in volume on *cooling* associated with the magnetic transformation which occurs at room temperature in alloys containing 36 per cent nickel. The inflection in the dilatation-temperature curve where, on heating, the coefficient of thermal expansion resumes a "normal" value, occurs at 240°C.

The low coefficient would clearly be of great interest in reducing thermal stresses at temperatures below 240°C. The modulus of elasticity of Invar is about 21×10^6, so that the "thermal stress modulus," $E\alpha$ at 15°C is $21 \times 1 = 21$, only one eighth of the next lowest value, that of titanium.

Unfortunately the mechanical strength of annealed Invar is only moderate (about 40 tons per sq. in.) and the density is high (specific gravity 8·1). The strength/weight ratio is thus about 5 compared with 10 for aluminium and even higher figures for magnesium and titanium alloys.

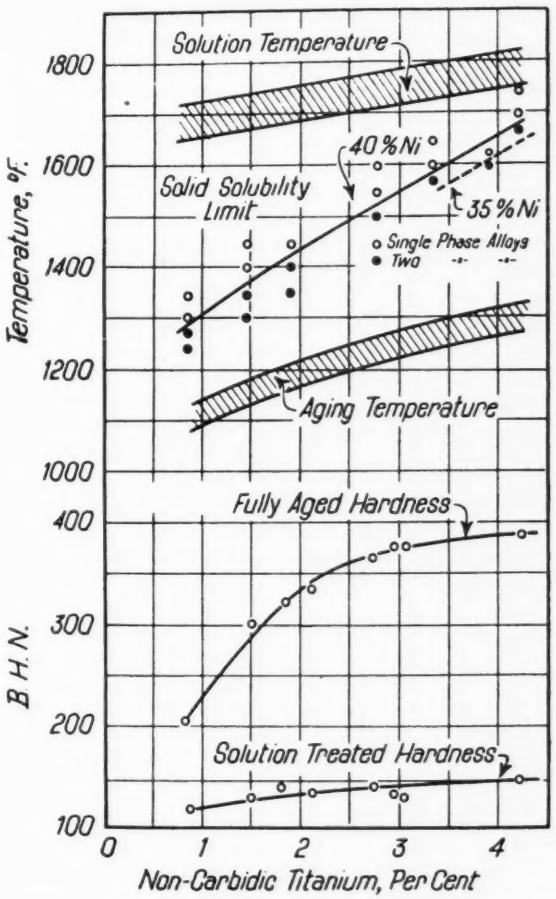


FIGURE 7. Solid solubility, heat treating temperatures and hardness of iron-nickel-titanium alloys containing approximately 40 per cent nickel. (Ref. 6).

It was observed in 1939 by Pilling and Talbot[6], of the International Nickel Company, that the addition of about 2 per cent of titanium to iron-nickel alloys of the Invar type made them capable of hardening by a two-stage heat treatment involving solution treatment at a high temperature (*e.g.* 950° to 1000°C), quenching, and precipitation-hardening at a temperature of 650° to 700°C (Fig. 7). The precipitate is believed to be the compound Ni_2Ti and 2 per cent titanium may therefore be expected to have the effect of diminishing by 5 per cent the effective nickel content in respect of the dilatational characteristics of the alloy. A typical alloy developed along these lines has the composition: nickel 41 per cent, titanium 2·4 per cent, iron remainder. After solution treatment the material is soft:

Brinell hardness 140, 0·2 per cent proof stress 18 tons per sq. in., ultimate tensile stress 40 tons in.^{2}, elongation 32 per cent.

Ageing for 9 hours at 675°C causes hardening to the following values:

Brinell hardness 330, 0·2 per cent proof stress 54 tons in.^{2}, ultimate tensile stress 74 tons in.^{2}, elongation 14 per cent.

By interposing a cold working operation between solution treatment and ageing, still higher values of strength and hardness can be achieved. As an example, 50 per cent cold working before ageing gives:

Brinell hardness 385, 0·2 per cent proof stress 74 tons in.$^{-2}$, ultimate tensile stress 87 tons in.$^{-2}$, elongation 5 per cent.

The modulus of elasticity in the age-hardened condition is 22 million lb. in.$^{-2}$

These mechanical properties bring the precipitation-hardened Invar into the same ranges of structural efficiency as the other materials which have been considered. Taking specific tensile strength

$$\left(\frac{\text{U.T.S.}}{d} \quad \frac{(\text{tons in.}^{-2})}{(\text{gm. c.c.})}\right)$$

the ratio is about 10, which is the same as the ratio for D.T.D.546, RR.58 and Rex 448 steel. For specific resistance to buckling the relatively high density brings the ratio, $\sqrt{E}/(d)$, down to about 0·7, *i.e.* in the same order as Nimonic 90.

Considering now thermal stress the advantage of the low coefficient of thermal expansion in the Invar-type alloys is seen. The average coefficient from 15° to 200°C is $3{\cdot}5 \times 10^{-6}$ per °C so that the thermal stress modulus is $22 \times 3{\cdot}5 = 77$, about one half of the value for titanium and one third of that for aluminium.

The thermal diffusivity, $K/\text{sp.ht.} \times \text{density}$, in c.g.s. units with $K = 0{\cdot}025$, sp.ht. $= 0{\cdot}11$ and density 8·1, is 0·03. This is the same as for the Nimonic alloys.

The quantity, Thermal stress modulus/Diffusivity, in c.g.s. units is therefore

$$\frac{1547 \times 3{\cdot}5}{0{\cdot}03}, \quad \textit{i.e.} \quad 180 \times 10^3$$

This puts the precipitation-hardened Invar close to titanium alloys as regards the magnitude of the thermal stress likely to be engendered under given conditions of heat input.

Reference to Fig. 8[7] shows how the inflection temperature for the iron-nickel alloys is raised as the nickel content increases from 36 per cent (or from 41 per cent in the presence of 2·4 per cent titanium). In the titanium-bearing alloy with 41 per cent nickel the inflection is at 220°C (430°F): with 46 per cent nickel it is at 340°C (640°F), but the coefficient of expansion, at $3{\cdot}2 \times 10^{-6}$ per °C in the 41 per cent nickel alloy is lower than in the 46 per cent nickel alloy, $6{\cdot}1 \times 10^{-6}$.

Figure 9[8] gives a comparison of nickel-iron alloys with a number of other materials in respect of total expansion from 15° to 500°C. The coefficient of expansion for aluminium is similar to that of the "N.M.C." steel included on the graph.

7. Structural Efficiencies at Elevated Temperatures

When the five classes of materials are compared in terms of structural efficiency at elevated temperatures the two criteria which we refer to in respect of proper-

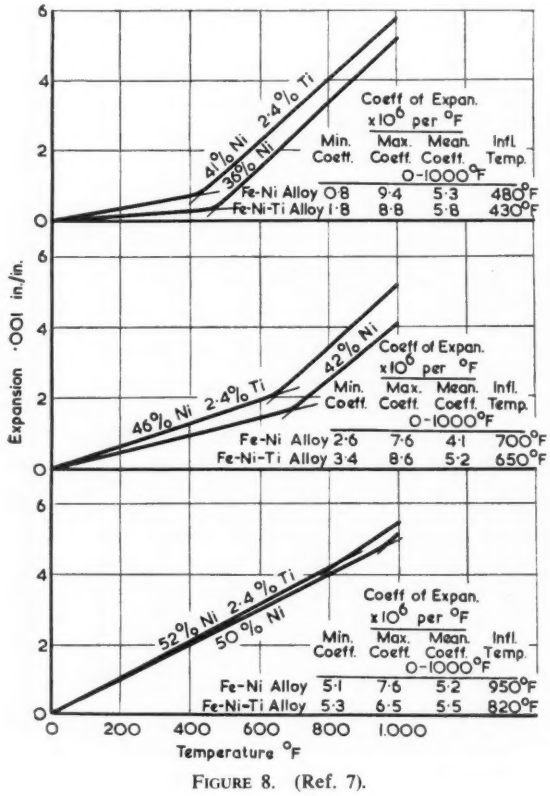

Expansion ·001 in./in.
Temperature °F
0 200 400 600 800 1,000
41% Ni 2·4% Ti
36% Ni
46% Ni 2·4% Ti
42% Ni
52% Ni 2·4% Ti
50% Ni
Coeff of Expan. x10⁶ per °F
Min. Coeff. | Max. Coeff. | Mean. Coeff. 0-1000°F | Infl. Temp.
Fe-Ni Alloy 0·8 | 9·4 | 5·3 | 480°F
Fe-Ni-Ti Alloy 1·8 | 8·8 | 5·8 | 430°F
Fe-Ni Alloy 2·6 | 7·6 | 4·1 | 700°F
Fe-Ni-Ti Alloy 3·4 | 8·6 | 5·2 | 650°F
Fe-Ni Alloy 5·1 | 7·6 | 5·2 | 950°F
Fe-Ni-Ti Alloy 5·3 | 6·5 | 5·5 | 820°F

FIGURE 8. (Ref. 7).

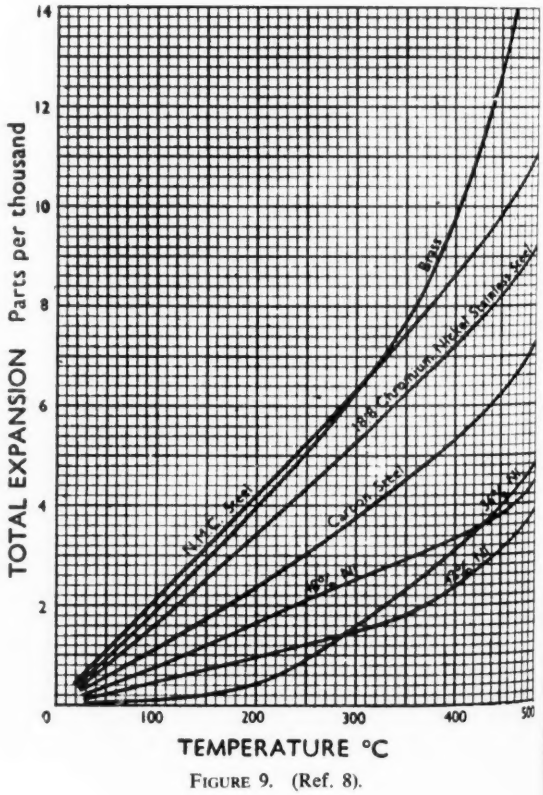


FIGURE 9. (Ref. 8).

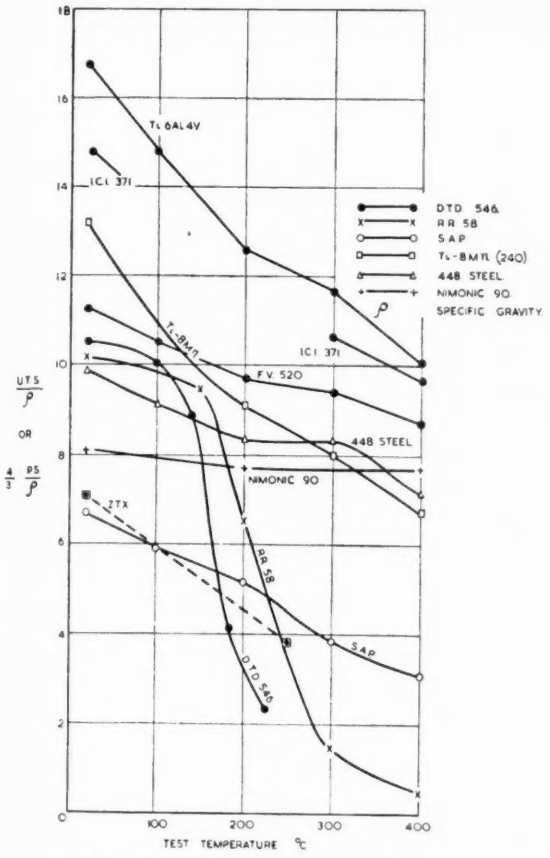


FIGURE 10. Structural efficiency in tension.

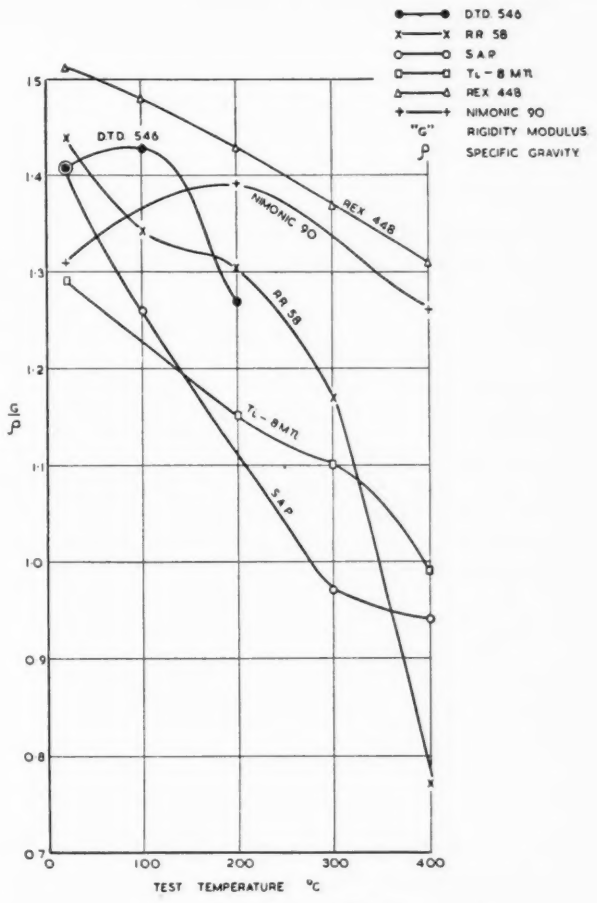


FIGURE 12. Structural efficiency in torsion.

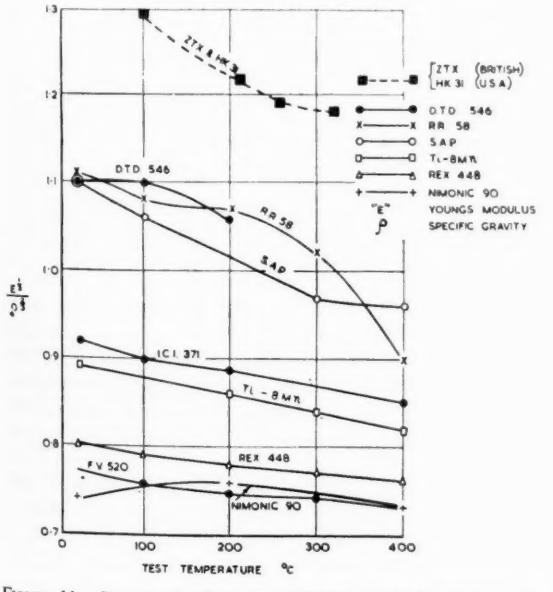


FIGURE 11. Structural efficiency of lightly loaded structures in compression. (Struts with high slenderness ratio).

ties at room temperature are still of major importance. These are the specific tensile strength, U.T.S./d, and the specific resistance to buckling. Data are available for the latter as the index for lightly loaded structures in compression, $E^{\frac{1}{2}}/d^{\frac{1}{2}}$, for struts with high slenderness ratio.

Figures 10, 11* and 12[2] summarise the relationships rather clearly.

(*a*) Structural efficiency in tension. On the basis of tensile strength at the elevated temperatures after 1,000 hours heating, the Duralumin family, L.65, D.T.D.546, and so on, is out of the running above 150°C; RR.58 holds its own up to 175°C; Rex 448 steel, 17-7PH and FV.520 steels and Nimonic 90 are superior to aluminium alloys above 175°C. The two titanium alloys I.C.I.371 (13 per cent tin, 2·75 per cent aluminium) and 6A14V are in a class by themselves above all the other materials. S.A.P., while inferior to conventional aluminium alloys at room temperature, is better above 225°C, and magnesium-thorium follows S.A.P. closely up to 300°C, but both S.A.P. and ZTX are far below the steels and titanium alloys.

*Based on data from G. Meikle.

(*b*) Structural efficiency in buckling of struts with high slenderness ratio. At all temperatures up to 400°C the density is the decisive factor; magnesium alloy ZTX is the best, then the aluminium alloys together, then titanium alloys and at the bottom the steels and nickel alloys.

(*c*) Structural efficiency in torsion. Here the rigidity modulus $G=E/2(1+\mu)$ where μ=Poisson's ratio, involves only the singular power of E and for the specific rigidity modulus we divide by the density. The steels and nickel alloys are the best from 150°C upwards.

8. Titanium Alloys and Precipitation-Hardening Steels

The most lively competition for the range of temperature from 150°C to 400°C at the present time is between the titanium alloys Ti6A14V and I.C.I.371 on the one hand and the precipitation-hardening chrome nickel steels, the U.S. Armco Steel Corporation's 17·7PH and Firth-Vickers FV520, on the other.

On the basis of tensile strength/weight ratio and also in respect of stiffness the titanium alloys are ahead of the steels at all temperatures to 400°C.

Sufficient creep data on a common basis are not yet available to permit a clear comparison of the titanium alloys and the steels. The nearest to a direct comparison from published results is:—

I.C.I. Ti371 (heat treated): at 400°C, under a stress of *30 tons in.*² a creep strain of 0·1 per cent occurred in a period longer than 300 hours.
at 300°C under a stress of *35 tons in.*² a creep strain of 0·1 per cent occurred in 200 hours.

Ti6A14V (heat treated): at 371°C (700°F) under a stress of *22·5 tons in.*² a creep strain of 0·1 per cent occurred in 112 hours.

FV.520 (precipitation-hardened steel): at 400°C under a stress of *25 tons in.*² a creep strain of 0·13 per cent occurred in 1,000 hours.

From this still meagre evidence one can conclude that the I.C.I. titanium alloy 371 (tin 13 per cent, aluminium $2\frac{3}{4}$ per cent) is better than Ti6A14V in creep resistance at 400°C. One would surmise also, but not with the same confidence, that at 400°C the I.C.I. titanium alloy is rather more resistant in creep than the FV.520 steel.

A recent communication[9] reports that the *specific stress to rupture* in creep at 427°C (800°F) in 100 hours of Ti6A14V is 65·5 per cent higher than that of 17-7PH steel. The same paper claims that the specific tensile strength and yield stress of Ti6A14V exceed by 25 per cent those of 17-7 steel from room temperature

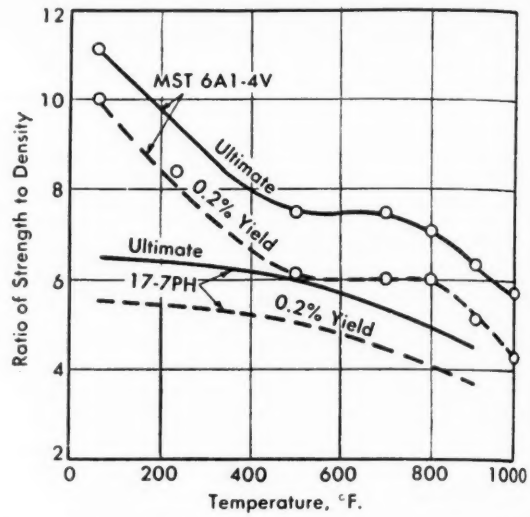


FIGURE 13(*a*). Tensile properties of heat treated MST 6Al-4V and 17-7 PH at elevated temperatures.

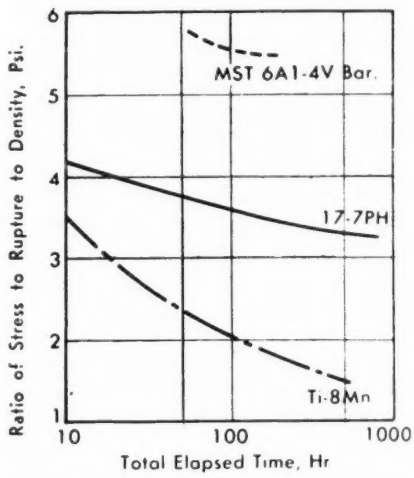


FIGURE 13(*b*). Comparison of stress-rupture properties at 800°F. (Ref. 9).

to 538°C (1000°F) (Fig. 13). In the foregoing comparisons the titanium alloy and the steel were both heat-treated to give a tensile strength of 80·5 tons in.² (180,000 lb. in.²) at the ordinary temperature.

The makers' advice regarding the upper limit of the operating temperature for the two families of material does not differ greatly. The American spokesman for Ti6A14V implies that he would put this limit at about 800°F (427°C). I.C.I. speak of their titanium alloy 371 being suitable for service at temperatures as high as 500°C.

Armco Steel Corporation do not claim a limit higher than 800°F (427°C) for 17-7 PH. Firth Vickers indicate that the limit may be 450°C or 500°C.

A choice between the I.C.I. titanium alloy 371, the American titanium alloy Ti6A14V and the precipitation hardened stainless steel 17-7PH or FV.520, may depend

in many cases on their relative ease or convenience in fabrication, including heat treatment.

I.C.I.371 is heat treated by first quenching from 1100°C. This gives a proof stress of 50 tons in.^{2} and an ultimate tensile stress of 60 tons in.^{2} Precipitation is carried out for one hour at 800°C, giving a proof stress of 60 tons in.^{2} and an ultimate tensile stress of 70 tons in.^{2}

The situation in respect of Ti6A14V and 17-7PH or FV.520 is the following. The first stage in the heat treatment puts the material into its softest condition by cooling rapidly from a relatively high temperature. For Ti6A14V this temperature is 850 to 950°C. The material is quenched from this temperature and then has a yield point of 46 to 66 tons in.^{2} and an ultimate tensile stress of 65 to 77 tons in.^{2}, *i.e.* a ratio (yield stress): (ultimate tensile stress) of 0·71 to 0·86.

For FV.520 the solution treatment comprises air-cooling from 1050°C. In this condition the yield point (0·1 per cent proof stress) is 21 tons in.^{2} and the ultimate tensile stress 55 tons in.^{2}, a ratio of (yield stress):(tensile stress) of 0·38.

These figures suggest that in the solution treated condition the stainless steel would be more easily formed by bending and pressing than the titanium alloy. Firth Vickers remark, however, that the softened FV.520 work hardens more rapidly than ordinary 18/8 stainless steel.

The precipitation-hardening treatments are:

for Ti6A14V: 8 to 24 hours at 480°C, and

for FV.520 : 2 hours at 700°C, air cool to room temperature, and precipitate 2 hours at 450° to 570°C.

The highest values of yield stress and ultimate stress obtainable in the two materials are:—

Ti6A14V: yield stress 72 tons in.^{2}, ultimate tensile stress 78 tons in.^{2}, elongation 16 per cent (ratio of yield/ultimate 0·92).

FV.520 : yield stress 70 tons in.^{2}, ultimate tensile stress 83 tons in.^{2}, elongation 23 per cent (ratio of yield/ultimate 0·84).

The Armco Corporation report the following for

17-7PH : yield stress 97 tons in.^{2}, ultimate tensile stress 102 tons in.^{2}, elongation 7 per cent (ratio of yield/ultimate 0·95).

The rationale of the heat treatment of FV.520 and 17-7PH is metallurgically interesting. It depends upon the fact that aluminium (and copper) are readily soluble in the form of iron-chromium-nickel alloy which is stable at high temperatures (*i.e.* austenite) and are relatively insoluble in the form to which the steel reverts at the ordinary temperature (ferrite-martensite). On rapidly cooling the steel from 1050°C it is retained for a short time in the austenitic form, but on resting at room temperature or at a lower temperature (*e.g.* −73°C) the austenite transforms within a period of 2 to 8 hours to martensite. The aluminium or copper is retained in supersaturated solid solution in the martensite: it is precipitated as Ni_3Al or a copper compound by heating to 450° to 570°C for two or three hours. The heating to 700°-800°C for two hours before the precipitation treatment is to precipitate carbide which, if left in solution, would stabilise the austenite and hinder its transformation into martensite.

It is unfortunate that these precipitation processes are accompanied by an appreciable change in density: it amounts to a decrease of two per cent in 17-7PH steel, corresponding to a linear expansion of 0·004 to 0·005 inch per inch. This means that final operations of machining, drilling and fitting must be carried out on the material in its fully hardened state.

The question is often asked: since titanium alloys cannot be of service for stressed components of kinetically heated structures when their temperature rises above about 450°C would it not be better, when changing from aluminium or magnesium alloys at around 200 or 250°C, to go directly to ferritic chromium steels, such as Rex 448, which are known to be suitable for operating temperatures as high as 650°C? Rex 448 has the composition:

Carbon 0·12, chromium 10·5, molybdenum 0·75, vanadium 0·15, and columbium 0·45 per cent.

It is a normal steel in the sense that it is fully hardened by quenching it in oil from 1150° to 1170°C. To put it into a condition suitable for engineering service it is tempered at 650°C to 700°C. After this treatment typical mechanical properties are:—

0·1 per cent proof stress : 55 tons in.^{2}
ultimate tensile stress : 67 tons in.^{2}
elongation : 20 per cent
(ratio: proof stress/ultimate tensile stress 0·82).

With the high quenching temperature of 1150° to 1170°C the extent of distortion must rule out any but the most rudimentary forming operations on the material before hardening. The major part of the shaping and forming must therefore be done on the steel in its final hardened condition. This is a serious disadvantage in comparison with FV.520 and 17-7PH which are soft after quenching from the high temperature and which can be formed while in this soft state and then hardened by heat treating to the modest temperatures of 450° to 570°C.

There would appear to be a strong case therefore for continuing to use precipitation hardening steels or titanium alloys for temperatures up to 450°C, turning over to ferritic alloy steels such as Rex 448 only at temperatures above this.

9. Selection of Materials for Kinetically Heated Structures

Looking back over this all too inadequate survey, I feel that the general principles to be followed in the selection of materials for kinetically heated structures might be summarised in the following way.

The conditions of service should be defined as precisely as possible in terms of temperatures likely to be attained by the various elements of the structure, the

duration of the exposure to these temperatures and the coincidence or separation of maximum loads and maximum temperatures.

With so much of this information as can be established, we must be prepared to design on the basis of using materials which are in a state of metallurgical instability under the conditions of service. This is not a new experience in the utilisation of materials in engineering: it applies to fatigue at the ordinary temperature, and to creep phenomena at elevated temperatures in gas turbines, steam turbines and high pressure steam fittings. This brings with it the conception of designing kinetically-heated components for a finite life. The only exception to this allowance for metallurgical instability is with sintered aluminium powder, S.A.P. It would appear to be worth while examining the possibility of making products analogous to S.A.P. based on magnesium and titanium.

In the range of temperature from 150° to 450°C there will be keen technical competition between the stronger heat-treated titanium alloys and precipitation-hardened stainless steels. The steels have an important advantage in their greater ease of working when in the "solution-treated" condition. In cases where a low coefficient of thermal expansion combined with high strength is of major importance, consideration might be given to the precipitation-hardening Invars. More information is required about their mechanical properties at elevated temperatures. There is a good prospect that these will be adequate at least up to 400°C, since the precipitation treatment is applied at the substantially higher temperature of 675°C.

I have given no attention in this Lecture to the need for resistance to corrosion and erosion in material for skins and for good fatigue properties at raised temperatures. Probably these factors would not lead to a different selection from one based on strength, stiffness and the physical characteristics already discussed. An exception is the magnesium-thorium alloy, which would certainly require special protection against corrosion and erosion.

Mention has been made of ferritic chromium steel (Rex 448) and the nickel-base materials Nimonic 80 and 90 and the American Inconel X. I did not discuss these at any length because I regard them as being more appropriate for operating temperatures higher than those which will be encountered at speeds up to a Mach number of 4.

This brings me to the expression of a philosophy.

Having defined as precisely as possible the most severe conditions for which provision must be made, the structural materials selected should be those which satisfy the requirements by the smallest reasonable margin. Experience teaches that generally in the engineering application of materials excessive coverage on mechanical strength almost always brings with it troublesome complications. These inconveniences may be primarily technical, for instance, difficult fabrication or undue notch-sensitivity, or economic, through cost or restricted availability. In the field of kinetically heated structures this argues in favour of dealing with the whole gamut of temperatures in as many separate successive subdivisions as can conveniently be accepted, selecting for each range the material just adequate for its limited demands.

ACKNOWLEDGMENTS

I have been greatly helped in the preparation of this lecture by a number of friends, who, while taking no responsibility for the views I have expressed, have placed at my disposal information and material. I should like to make special acknowledgment to:

Mr. G. Meikle, of the Royal Aircraft Establishment, Farnborough, Dr. N. P. Allen, of the National Physical Laboratory, Dr. C. Sykes, of Thomas Firth and John Brown Ltd., Dr. R. T. Parker, of Aluminium Laboratories Ltd., Dr. W. M. Doyle, of High Duty Alloys Ltd., Dr. E. A. Bloch, of Aluminium Industrie, Neuhausen, Switzerland, Mr. L. W. Johnson, of The Mond Nickel Co. Ltd., Dr. N. P. Inglis, of Imperial Chemical Industries Ltd., Metals Division, and the Librarian and the Photographic Department of the College of Aeronautics.

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Flutter of Wings with Localised Masses

by

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SUMMARY: From a general consideration of the available data on the flutter of wings with localised masses certain deductions are made as to the possible types of flutter that can occur. On the basis of these deductions it is shown that there is an optimum choice of modes for use in flutter calculations for wings with localised masses. These modes are obtained with artificial constraints imposed on the wing at the localised mass section fixing the wing at this section in translation and/or pitch. It is deduced that for certain mass locations types of flutter are obtained that are insensitive to increase of localised mass, beyond a certain value, with flutter speeds considerably greater than that of the fixed root bare wing. It is also deduced that for the majority of aircraft configurations the maximum flutter speeds for these types of flutter will be realised when the localised mass is in the region of two-thirds semi-span from the root. A limited theoretical investigation is made for a rectangular unswept uniform wing with symmetric and antisymmetric body freedoms, to illustrate and confirm the conclusions derived from general considerations. At the same time the investigation shows that an ill placed localised mass can reduce the wing flutter speed to a very low value.

1. Introduction

The mounting of localised masses, such as engines, missiles, fuel tanks, and so on, on aircraft wings is a common feature of present-day aircraft. These localised masses are often considerably greater than the mass of the wing itself and can have a considerable effect on the wing flutter speed, leading either to an increase or decrease in flutter speed depending on the mass parameters and the location of the mass.

Various experimental and theoretical investigations of the flutter of wings with localised masses have been made, but in general the experiments are confused by the many different types of flutter that occur, and the theoretical work suffers from an uncertainty as to the type and number of distortion modes that must be included in the calculations to obtain a representative solution.

In what follows the available data is analysed in an attempt to resolve these difficulties. Simple relationships are deduced for defining the different types of flutter that can occur for particular configurations, including configurations with body freedoms. These relationships also determine the types of modes that are of significance in the flutter and the optimum number of modes that will provide representative flutter solutions.

A limited theoretical investigation is made of the flutter of an unswept uniform wing with localised masses, and with symmetric and antisymmetric body freedoms. The results of this investigation serve to confirm the deductions made and demonstrate the powerful effects of localised masses on wing flutter speeds.

2. Experimental data

A large proportion of the available data on the flutter of wings with localised masses is devoted to investigations on fixed root wings[1, 2, 3, 4, 5, 6]. The most systematic and comprehensive of these investigations are those of Lambourne and Weston[1] (1944) and Gaukroger[2] (1953). Systematic data for wings with body freedoms are provided almost entirely by Gaukroger[7, 8, 9, 10, 11] and are mainly concerned with effects of a central localised mass (*i.e.* the aircraft fuselage) on flutter. However, the most recent of Gaukroger's tests[11] is an investigation of effects on flutter of a combination of central localised mass and outboard localised mass.

On analysing these data it is apparent that one outstanding feature is common to the different types of flutter obtained for all the configurations tested. It appears that the ultimate effect of a localised mass on wing flutter is to lead to a more or less abrupt change in the type of flutter that occurs, and is associated with a change in the motion of the mass. Flutter involving initially large motions of the mass changes abruptly with localised mass variation to a type of flutter where the motion of the mass is small. The transition from one type of flutter to another may be associated with a change from large to small translation of the mass or from large to small pitch, or to a condition where both translation and pitch are small.

In what follows it is assumed that to a first approximation these flutter transitions can be regarded as transitions from large to zero mass movement, and the zero condition then corresponds to fixing the loading section in translation and/or pitch. It is further assumed that the flutter of the wing under this artificial constraint can be represented by a coupling of the fundamental bending and torsion modes of the constrained wing. These features are then seen to provide a useful guide to the type of wing modes of significance in the flutter of wings with localised masses, and they also provide a ready means of identifying the different types of flutter that can occur. This is of particular value in theoretical investigations of the flutter of wings with localised

Received 31st December 1956.

masses, where it is important that an adequate number of degrees of freedom are included for a representative flutter solution, while at the same time the number must be kept to a minimum because of limitations in computing equipment.[13]

2.1. FIXED ROOT WING

Consider first the case of a fixed root wing with a single localised mass. For this case application of the foregoing principles leads to four basic types of flutter for different combinations of mass movements, as follows:—

Type (*a*)—Large motion of the mass in translation and pitch.

Type (*b*)—Zero translation of the mass, large pitch motion.

Type (*c*)—Large translation of the mass, zero pitch motion.

Type (*d*)—Zero translation, zero pitch motion of the mass.

When the centre of gravity of the mass coincides with the wing flexural axis, zero displacement of the mass also implies zero displacement of the wing at that section. For this condition it is apparent that the following five modes are associated with the different types of flutter (*see* Fig. 1).

Displacement modes		*Twist modes*	
1	Bending with root fixed.	3	Torsion with root fixed.
2	Bending with root and mass section fixed.	4(*a*)	Torsion of inner wing with root and mass section fixed.
		4(*b*)	Torsion of outer wing with mass section fixed.

These are uncoupled modes for the unloaded wing, with modes 2 and 4 obtained by imposing an arbitrary constraint at the mass loading section. Mode 2 (bending) is continuous since the wing is constrained in displacement only, rotations in the wing plane being unrestricted. However, for modes 4 (torsion) in which the wing is constrained in pitch there are two distinct torsion modes at different frequencies for the inboard and outboard parts of the wings.

The four different types of flutter are then provided by the following combinations of modes

Type (*a*) flutter—modes 1 and 3.
Type (*b*) flutter—modes 2 and 3.
Type (*c*) flutter—modes 1 and 4.
Type (*d*) flutter—modes 2 and 4.

When the centre of gravity of the mass does not coincide with the wing flexural axis, zero displacement of the mass does not necessarily imply zero displacement of the wing at the mass section, and conversely. The following modal combinations are then required:—

Type (*a*)—modes 1 and 3.
Type (*b*)—modes 1, 2 and 3.
Type (*c*)—modes 1 and 4.
Type (*d*)—modes 2 and 4.

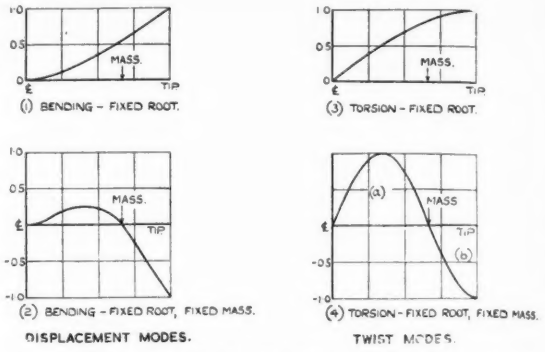

FIGURE 1. Fixed root—relevant wing modes.

However, it follows that all the different types of flutter that can occur for this particular wing-mass combination are covered by a flutter calculation using the above five modes.

Furthermore there are certain general conclusions that can be inferred for the different types of flutter that occur.

(1) To obtain a transition from one type of flutter to another their flutter speeds must be of the same order.

(2) The flutter speeds for types (*a*) and (*b*) flutter will generally be lower than for types (*c*) and (*d*) flutter since the latter are associated with torsion modes of higher frequency. Consequently for a transition from types (*a*) or (*b*) to types (*c*) or (*d*) the localised mass must be so placed as to provide a favourable increase in speed for types (*a*) or (*b*) flutter.

For example, suppose a mass is added to the wing forward of the flexural axis. This contributes towards massbalance of the wing and might be expected to increase the flutter speed. At the same time the mass imposes a restraint against displacement of the wing at that section, and its moment of inertia about the wing flexural axis imposes a restraint in torsion. The flutter is initially type (*a*) and one might therefore expect a transition to types (*b*), (*c*) or (*d*) as the mass is increased. If the transition is to type (*b*) then further increase in mass can only affect the restraint to wing twist and the next transition will be to type (*d*). Similarly if the first transition is to type (*c*) the next transition will be to type (*d*). Type (*d*) flutter is insensitive to mass increase.

It follows that within the practical range of localised mass variations transitions from type (*a*) to other forms of flutter are less likely to occur for masses added aft of the wing flexural axis, than for masses added forward, since an increase in wing flutter speed is less likely to be obtained. Accordingly fewer modes need be considered in calculations for aft positions of the mass than for forward positions.

Types (*c*) and (*d*) flutter will generally consist primarily of a coupled oscillation between the bending mode and the torsion mode 4(*a*) or 4(*b*) of lowest frequency (provided the wing is uniform) flutter speed increasing as this frequency increases. The flutter speeds associated with these types of flutter might thus be expected to reach a maximum when these torsion mode

frequencies are equal. This condition is realised when the mass loading section coincides with the chordwise nodal line position for the first overtone torsion mode of the bare wing. For a rectangular, unswept, uniform wing this nodal line is at two-thirds semi-span from the wing root, and in practice is in much this same position for a wide variety of plan forms.[14]

The chordwise nodal line location for the first overtone torsion mode of the bare wing is therefore an optimum position for mounting a localised mass, if the mass parameters are such that transition to types (*c*) or (*d*) flutter obtains.

Comparing these deductions with experimental evidence the following is found:—

(1) Transition from type (*a*) to type (*b*) flutter is obtained in two of the investigations.[1, 3]

(2) Transitions from type (*a*) to type (*c*) flutter is not obtained in any of the investigations.

(3) Transition to type (*d*) flutter is obtained in five of the investigations.[1, 2, 3, 4, 5]

It appears that transition to type (*d*) flutter is the most likely occurrence and transition to type (*c*) flutter the least likely. This is not unreasonable since type (*b*) flutter will usually be associated with a localised mass of large mass but small moment of inertia, a possible but unlikely combination; type (*c*) with a localised mass of small mass, large moment of inertia, a very unlikely combination; and type (*d*) with a localised mass of large mass and moment of inertia, the most likely combination.

Furthermore, in all cases where type (*d*) flutter has been obtained there are clear indications that the flutter speed associated with this type of flutter reaches a maximum when the localised mass is in the region of two-thirds span. In the tests of Nelson and Tommassoni[3] the wing flutter speed with a mass at this section was 2·5 times that for the bare wing, and in Gaukroger's tests[2] it was greater than twice the bare wing speed, the actual flutter speed being outside the tunnel speed range.

2.2. ROLL FREEDOM—CENTRAL MASS, NO OUTBOARD MASSES

Here there are two possible types of flutter, namely

Type (*e*) flutter—Large roll motion of the central mass.

Type (*a*) flutter—Zero roll motion of the central mass

and the three modes associated with these types of flutter are:—

Displacement modes	*Twist modes*
(1) Rigid body roll.	(3) Torsion with root fixed.
(2) Bending with root fixed.	

Type (*e*) flutter is a combination of modes 1 and 3 and type (*a*) flutter has already been described.

The two types of flutter for this wing mass combination are therefore covered by a flutter calculation using these three modes.

One would expect little change in either flutter speed or frequency between flutter involving modes 1 and 3 and modes 2 and 3 since, in general, the flutter characteristics are controlled more by the torsion than the displacement modes. Also, one would expect type (*e*) flutter to be associated with small roll inertia of the central mass and type (*a*) flutter with large roll inertia.

The experimental results of Gaukroger[7, 8] confirm these deductions.

2.3. ROLL FREEDOM—CENTRAL MASS AND OUTBOARD WING MASS

For this arrangement there are many more types of flutter since the central mass and the wing mass can have zero motions together and independently. The types of flutter are:—

Type (*e*) flutter—Translation and pitch of the wing mass, roll of the central mass.

Type (*a*) flutter—Translation and pitch of the wing mass, zero roll of the central mass.

Type (*f*) flutter—Zero translation, large pitch of the wing mass, roll of the central mass.

Type (*b*) flutter—Zero translation, large pitch of the wing mass, zero roll of the central mass.

Type (*g*) flutter—Large translation, zero pitch of the wing mass, roll of the central mass.

Type (*c*) flutter—Large translation, zero pitch of the wing mass, zero roll of the central mass.

Type (*h*) flutter—Zero translation, zero pitch of the wing mass, roll of the central mass.

Type (*d*) flutter—Zero translation, zero pitch of the wing mass, zero roll of the central mass.

The relevant modes are (*see* Fig. 2):—

Displacement modes		*Twist modes*	
1	Rigid body roll.	5	Torsion with root fixed.
2	Bending with root fixed.	6(*a*)	Torsion of inner wing with root fixed and mass fixed.
3	Bending with root free, mass fixed.	6(*b*)	Torsion of outer wing with mass fixed.
4	Bending with root fixed, mass fixed.		

The modes associated with the three new types of flutter (*f*), (*g*), (*h*), are:—

Type (*f*)—modes 3 and 5.

Type (*g*)—modes 1 and 6.

Type (*h*)—modes 3 and 6.

All eight types of flutter are covered by flutter calculations using the above seven modes. It should be noted that modes 6 reduce to one single mode when the localised mass loading section is at the tip or when it coincides with the chordwise nodal line location for the (antisymmetric) overtone torsion mode of the bare wing.

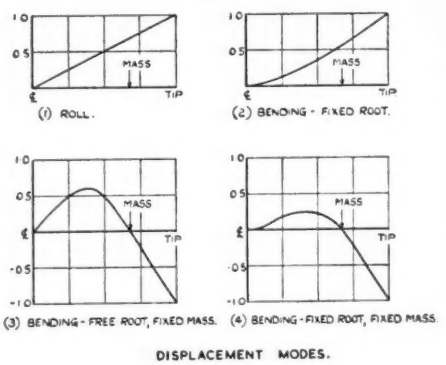

FIGURE 2. Roll freedom—relevant wing modes.

The flutter speeds for types (*g*), (*c*), (*h*), (*d*) flutter will in general be higher than those for types (*e*), (*a*), (*f*) and (*b*). Further, following the same argument as for the fixed root wing, transitions from types (*e*) and (*a*) flutter to other types are less likely to occur for c.g. positions of the localised mass aft of the wing flexural axis and fewer modes then need be considered in the flutter calculations.

As for the fixed root wing the flutter speeds for types (*g*), (*c*), (*h*), (*d*) flutter are at maximum when the localised mass loading section coincides with the chordwise nodal line for the antisymmetric overtone torsion mode of the bare wing, *i.e.* when the mass is in the region of two-thirds semi-span from the root.

Some recent tests by Gaukroger and Chapple[11] provide systematic data with which these deductions can be compared. They positively obtain types (*e*), (*f*), (*h*) and (*d*) flutter and might possibly have obtained types (*a*) and (*b*) flutter without identifying them since their flutter speeds and frequencies would be close to those for types (*e*) and (*f*) respectively. Types (*g*) and (*c*) flutter were not obtained and they are unlikely to be obtained in practice since they are generally associated with small mass, large moment of inertia values of the localised mass. The tests also show that the optimum loading section for types (*h*) and (*d*) flutter is in the region of two-thirds semi-span, the flutter speeds being higher than twice the bare wing speed.

2.4. PITCH AND TRANSLATION FREEDOM—CENTRAL MASS, NO OUTBOARD MASS

There are four possible types of flutter for this arrangement, namely:—

Type (*j*)—Translation and pitch of the central mass.
Type (*k*)—Zero translation, large pitch of the central mass.
Type (*l*)—Large translation, zero pitch of the central mass.
Type (*a*)—Zero translation, zero pitch of the central mass

and the modes associated with these types of flutter are:—

Displacement modes	*Twist modes*
1 Rigid body translation.	3 Rigid body pitch.
2 Bending with root fixed.	4 Torsion with root fixed.

The modes associated with the new types of flutter (*j*), (*k*), and (*l*) are:—

Type *j*—modes 1 and 3.
Type *k*—modes 2 and 3.
Type *l*—modes 1 and 4.

and all four types of flutter are covered in flutter calculations using the above four modes.

The flutter speeds for types (*l*) and (*a*) flutter will generally be higher than for types (*j*) and (*k*) flutter. Further, a transition from the former to the latter is less likely for c.g. positions of the central mass forward of the flexural axis.

Comparing with experimental data we find that Gaukroger obtains only types (*k*) and (*a*) flutter in his tests[9, 10] while Lambourne[12] obtains types (*j*), (*k*) and (*a*). Type (*j*) is the longitudinal dynamic stability oscillation, involving no structural distortion, and is unlikely to occur for a stable aircraft. Gaukroger only investigated stable configurations but Lambourne included some that were unstable. In no case has type (*l*) flutter been obtained but this is associated with a central body of large inertia and small mass, an unlikely practical combination.

2.5. PITCH AND TRANSLATION FREEDOM—CENTRAL MASS AND OUTBOARD WING MASS

Proceeding as before the 16 possible types of flutter are:—

Type (*j*)—Translation and pitch of outboard mass and central mass.
Type (*k*)—Translation and pitch of outboard mass, zero translation large pitch of the central mass.
Type (*m*)—Zero translation, large pitch of outboard mass, translation and pitch of the central mass.
Type (*n*)—Zero translation, large pitch of outboard mass, zero translation, large pitch of the central mass.

Type (*l*)—Translation and pitch of outboard mass, large translation zero pitch of central mass.
Type (*a*)—Translation and pitch of outboard mass, zero translation, zero pitch of central mass.
Type (*o*)—Zero translation, large pitch of outboard mass, large translation zero pitch of central mass.
Type (*b*)—Zero translation, large pitch of outboard mass, zero translation, zero pitch of central mass.

Type (*p*)—Large translation, zero pitch of outboard mass, translation and pitch of central mass.

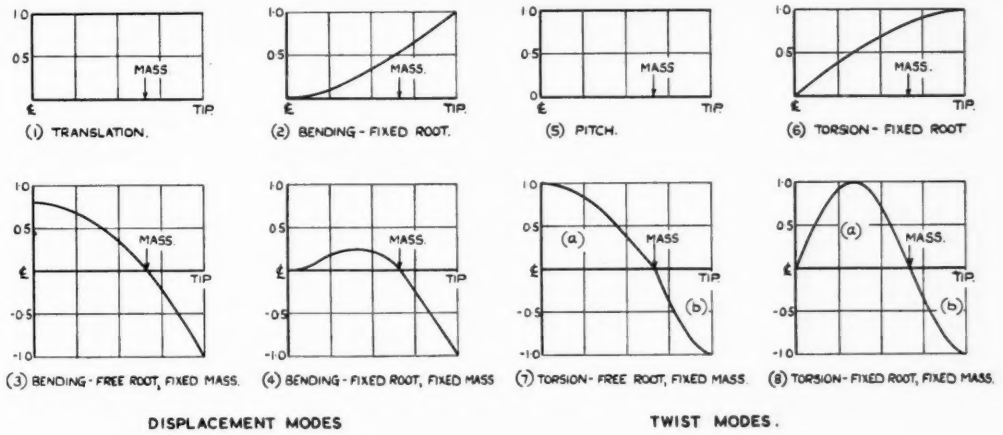

FIGURE 3. Pitch and translation freedom—relevant wing modes.

Type (*g*)—Large translation, zero pitch of outboard mass, zero translation, large pitch of central mass.

Type (*r*)—Zero translation, zero pitch of outboard mass, translation and pitch of central mass.

Type (*s*)—Zero translation, zero pitch of outboard mass, zero translation, large pitch of central mass.

Type (*t*)—Large translation, zero pitch of outboard mass. Large translation, zero pitch of central mass.

Type (*c*)—Large translation, zero pitch of outboard mass, zero translation, zero pitch of central mass.

Type (*u*)—Zero translation, zero pitch of outboard mass. Large translation, zero pitch of central mass.

Type (*d*)—Zero translation, zero pitch of outboard mass, zero translation, zero pitch of central mass.

The modes associated with these types of flutter are: (*see* Fig. 3).

Displacement modes

1 Rigid body translation.
2 Bending with root fixed.
3 Bending with root free, mass fixed.
4 Bending with root fixed, mass fixed.

Twist modes

5 Rigid body pitch.
6 Torsion with root fixed.
7(*a*) Torsion of inner wing, root free, mass fixed.
7(*b*) } Torsion of outer
8(*b*) } wing, mass fixed.
8(*a*) Torsion of inner wing, root fixed, mass fixed.

Of these 16 types of flutter there are eight new types and the modes associated with them are:—

Type (*m*)—modes 3 and 5.
Type (*n*)—modes 4 and 5.
Type (*o*)—modes 3 and 6.
Type (*p*)—modes 1 and 7.
Type (*q*)—modes 2 and 7.
Type (*r*)—modes 3 and 7.
Type (*s*)—modes 4 and 7.
Type (*t*)—modes 1 and 8.
Type (*u*)—modes 3 and 8.

All 16 types of flutter are covered by calculations using the above nine modes.

The lowest flutter speeds will generally be associated with types (*j*), (*k*), (*m*), (*n*) increasing to types (*l*), (*a*), (*o*), (*b*), then to types (*p*), (*q*), (*r*), (*s*), the highest speeds being achieved for types (*t*), (*c*), (*u*), (*d*) flutter. The latter two groups are both associated with coupled oscillations between displacement modes and torsion of either the inboard or outboard parts of the wing and, following the reasoning of Section 2.1, it follows that maximum flutter speeds for these types of flutter are obtained for some particular optimum location of the mass. For types (*t*), (*c*), (*u*), (*d*) flutter where both central mass and outboard mass sections are fixed in pitch, the optimum section will be the chordwise nodal line for the overtone torsion mode of the fixed root wing, and will be in the region of two-thirds semi-span from the root. However, for types (*p*), (*q*), (*r*), (*s*) flutter there is large movement of the central mass in pitch, the torsion mode being of the fundamental free-free type. The latter types of flutter will usually be associated with small moment of inertia of the central fuselage. For the particular case of a uniform unswept rectangular wing with zero central inertia the chordwise nodal line for the fundamental free-free torsion mode is at half semi-span from the root. As the central inertia is increased this optimum section moves inboard.

It is of interest to consider how many of the 16 possible types of flutter are likely to occur in practice and the configurations with which they are associated.

Type (*j*) will not occur for a stable aircraft.

Types (*l*), (*o*), (*p*), (*q*), (*t*), (*c*), (*u*), are unlikely since they involve the unlikely combination of a small mass, large moment of inertia for either the central mass or the outboard mass.

Types (*k*), (*n*), (*q*), (*s*), are associated with a large central mass of small moment of inertia, an unlikely combination although it may occur on a flying wing type aircraft.

Types (*m*), (*p*), (*r*), are associated with a central mass of small mass and moment of inertia, conditions only found in a twin-boom or flying wing type aircraft.

Types (*m*), (*o*), (*b*), are associated with a large outboard mass of small moment of inertia, an unlikely combination.

There remain types (*a*) and (*d*) flutter, which are the types of symmetric flutter most likely to be met with for aircraft and missiles where the mass and moment of inertia of the central mass are high. For these types of flutter fixed root wing modes may be adequate in calculations (*see* Section 2.1). The only available experimental evidence to compare with these deductions is provided by some recent tests by Gaukroger and Chapple.[11] For their tests the central mass was invariably of a fairly large mass and moment of inertia, and the wing mass combinations investigated were also of large mass and moment of inertia. In consequence only types (*a*) and (*d*) flutter were obtained. However, the flutter curves did not follow those for the fixed root wing in all cases, indicating that the mobility of the central mass was of importance for their configuration. The tests also showed that an optimum position of the outboard mass for type (*d*) flutter was probably between 0·5 and 0·75 semi-span from the root.

3. Theoretical Investigation

There is little theoretical evidence on the flutter of wings with localised masses that is of value for the purposes of comparison with the above deductions. Minhinnick and Yarwood[15] and Frazer[16] have investigated the effects of localised mass on the flutter of a fixed root wing, but in neither case do the modes used agree with those suggested above, although by linear combinations of them reasonable approximations to those suggested here can be obtained. Minhinnick and Yarwood[15] obtain a transition from type (*a*) to type (*d*) flutter with simultaneous variation of the mass and moment of inertia of a localised mass forward of the wing flexural axis, and Frazer[16] obtains a transition from type (*a*) to type (*b*) flutter with variation of localised mass value for a position on the flexural axis.

More recently Seal and Broadbent[17] have investigated the effects of a tip mass on the flutter of a fixed root wing for comparison with Gaukroger's results[2]. The modes used are similar to those suggested here, but two higher order modes are included. They obtain transition from type (*a*) to type (*d*) flutter, and reasonable quantitative agreement with experiment.

Theoretical work for wings with body freedoms is even more limited. Houbolt[18] has investigated the case of roll freedom with central mass and no outboard mass and obtains a transition from type (*e*) to type (*a*) flutter as predicted here and as confirmed by experiment[7, 8]. Seal[19] has investigated the antisymmetrical flutter of an aircraft with an outboard mass, but she considers only three modes (two aircraft modes and roll) as compared with seven which are required ideally (Section 2.3). None of the transitions predicted here are obtained. There appears to be no systematic theoretical data for the case of symmetric freedoms.

TABLE I

Wing details

Wing span (tip to tip)	=4 ft.
Wing chord	=1 ft.
Wing flexural axis	=0·25 ft. aft of wing leading edge
Wing inertia axis	=0·45 ft. aft of wing leading edge
Radius of gyration of wing section	=0·316 ft. about flexural axis
Wing weight	=0·0476 slugs per ft.
Wing pitch inertia	=0·019 slugs ft.^{2} about flexural axis
Wing roll inertia	=0·273 slugs ft.^{2} about centre line
Uncoupled, fixed root, wing bending frequency	=16 cycles per sec.
Uncoupled, fixed root, wing torsion frequency	=50 cycles per sec.

It is apparent that systematic theoretical data covering all the arrangements considered here is lacking, and to bridge this gap a limited theoretical investigation has therefore been made. The investigation is of an unswept uniform rectangular wing with localised mass and with symmetric and antisymmetric body freedoms. The main details of the wing are given in Table I. Two-dimensional incompressible flow theory is used throughout for a fixed frequency parameter of 0·6.

In all cases flutter speeds and frequencies are expressed as ratios to the speed V_B and frequency ω_B respectively obtained from a binary flutter calculation for the bare wing using modes of fixed root bending and torsion. These two degrees of freedom are included in the calculations for all the cases considered.

3.1. FIXED ROOT

For the fixed root wing two cases are considered; namely, that of a localised mass at the tip and that of a localised mass at two-thirds semi-span from the root. For these particular spanwise sections modes 4 (Section 2.1 and Fig. 1) reduce to one single mode and a quaternary flutter calculation is required. The modes used are the uncoupled flexural and torsional modes for the bare wing, with appropriate constraints at the localised mass section for modes 2 and 4 (Fig. 1).

3.1.1. *Localised Mass at the tip*

The particular cases of a concentrated mass at the wing leading edge and on the wing flexural axis at 0·25 chord are considered. The results are shown in Fig. 4.

With the mass at the leading edge (Fig. 4(*a*)) the flutter for small mass value is initially of type (*a*). As the mass is increased the flutter speed increases and the frequency decreases, until the mass is about 0·16 of the wing mass when a transition to type (*d*) flutter occurs. The speed then falls with further increase in mass tending to an asymptotic value that is about 1·65 times the flutter speed for the bare wing.

The binary solution using the fixed root bending and torsion modes, 1 and 3 (Fig. 1), follows the quaternary solution quite closely up to the point where transition

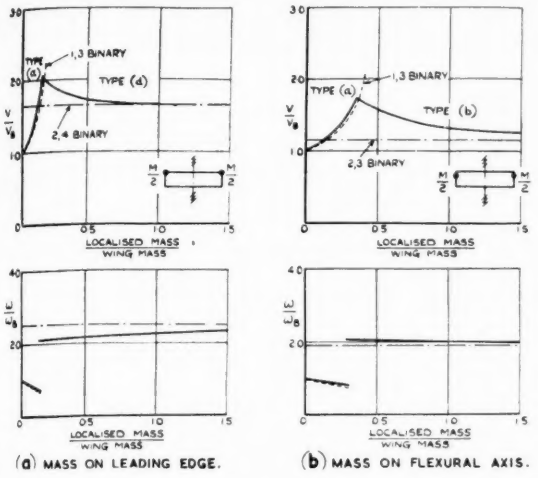

FIGURE 4 (*a* and *b*). Fixed root flutter—mass at the tip.
$V_B = 538$ ft./sec.
$\omega_B = 236$ rads./sec.

to type (*d*) flutter occurs, but the latter branch of the curve is only obtained from the quaternary solution. However, the asymptotic value of the flutter speed for type (*d*) flutter is predicted closely by a binary flutter calculation using modes 2 and 4.

With the mass on the wing flexural axis (Fig. 4(*b*)) the flutter is again initially of type (*a*) and as the mass is increased the flutter speed increases, until the mass is about 0·35 of the wing mass when a transition to type (*b*) flutter occurs. The speed then falls with increasing mass, tending to an asymptotic value that is about 1·25 times the flutter speed for the bare wing. The binary solution using modes 1 and 3, predicts the region for type (*a*) flutter closely, and the binary using modes 2 and 3 defines the flutter speed asymptote for large mass values. Further, the ternary solution using modes 1, 2 and 3 is everywhere within ± 3 per cent of the quaternary solution, indicating that mode 4 is unimportant in this case. This provides some confirmation of the suggestion that fewer modes are required for an adequate flutter solution with aft positions of the mass.

3.1.2. *Localised Mass at Two-thirds Semi-span*

The results are shown in Fig. 5. With the mass on the wing leading edge (Fig. 5(*a*)) a transition from type (*a*) to type (*d*) flutter occurs when the mass is about 0·5 of the wing mass. The flutter speed for type (*d*) flutter is about 2·7 times that for the bare wing, which is much greater than for the tip mass case.

The effect of variation of the radius of gyration of the mass has been investigated and this is seen to have a powerful effect on the flutter speeds for type (*a*) flutter but does not affect either the mass ratio for transition or the flutter characteristics of type (*d*) flutter.

The type (*a*) flutter region is predicted closely by the binary solution using modes 1 and 3, and the binary using modes 2 and 4 defines the asymptote for type (*d*) flutter. Modes 1, 2, 3 and 4 are all necessary for the complete solution.

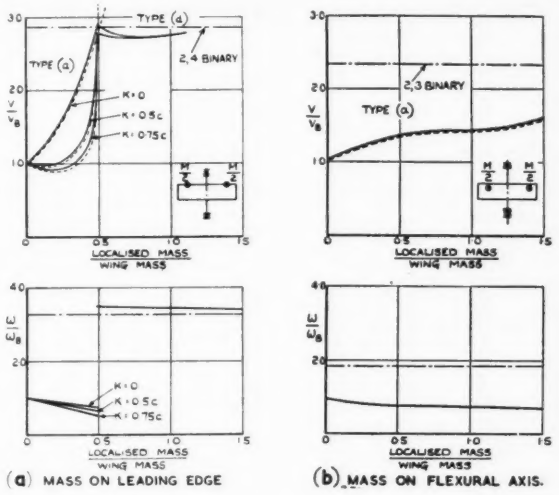

FIGURE 5 (*a* and *b*). Fixed root flutter—mass at two-thirds semi-span from root.
$V_B = 538$ ft./sec.
$\omega_B = 236$ rads./sec.

For a mass on the flexural axis (Fig. 5(*b*)) the flutter is of type (*a*) throughout, the flutter speed increasing and the frequency decreasing with increasing mass value. Presumably, for a sufficiently large mass a transition to type (*b*) flutter will ultimately occur but the asymptote for this type of flutter, defined by the binary using modes 2 and 3, is well above the speed for type (*a*) for the range of mass variation considered here. For the range of variation considered the whole flutter region is predicted very closely by the binary solution using modes 1 and 3.

3.2. ROLL FREEDOM

3.2.1. *Central Mass, No Outboard Masses*

The modes used in the calculation are wing roll, fixed root bending and fixed root torsion of the bare wing (modes 1, 2, and 5 of Fig. 2), and the effect of variation in the rolling moment of inertia of the central mass is investigated.

The results are shown in Fig. 6. The flutter is initially of type (*e*) and a transition to type (*a*) flutter occurs when the roll inertia of the central mass is

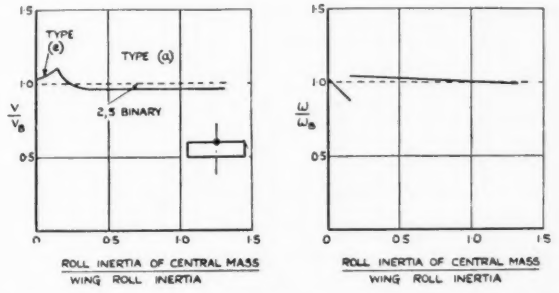

FIGURE 6. Roll freedom—central mass—no outboard masses.
$V_B = 538$ ft./sec.
$\omega_B = 236$ rads./sec.

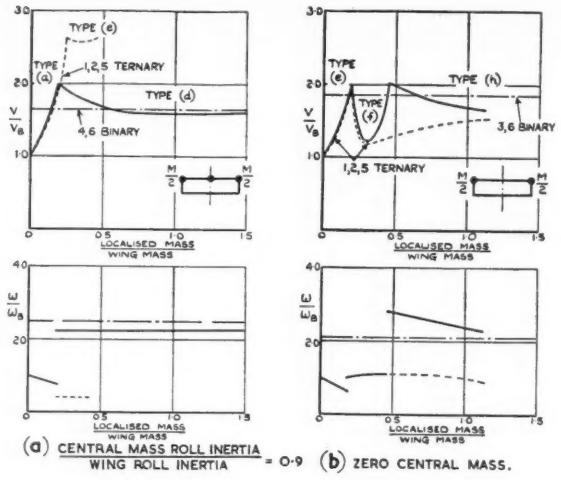

FIGURE 7 (*a* and *b*). Roll freedom—central mass—outboard mass on leading edge at wing tip.
$V_B = 538$ ft./sec.
$\omega_B = 236$ rads./sec.

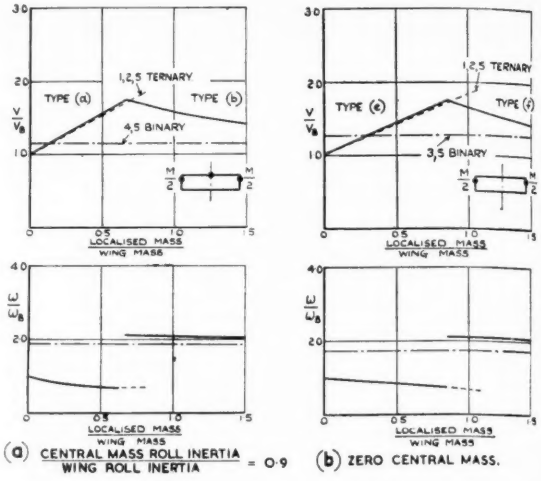

FIGURE 8 (*a* and *b*). Roll freedom—central mass—outboard mass on flexural axis at tip.
$V_B = 538$ ft./sec.
$\omega_B = 236$ rads./sec.

increased to about 0·15 of that of the wing. The binary solution using modes 2 and 5 provides the asymptote for type (*a*) flutter, but all three modes are necessary for the complete solution.

3.2.2. *Central Mass and Outboard Mass*

The cases are considered of an outboard mass at the wing tip and at two-thirds semi-span from the root. For each of these cases the two chordwise mass positions on the leading edge and on the flexural axis are considered, together with the conditions of large roll inertia of the central mass (0·9 of the wing roll inertia) and zero roll inertia of the central mass. These two central mass conditions ensure that the initial flutter with small outboard mass is of type (*a*) in one case and type (*e*) in the other.

The six modes required in the calculations for these cases are shown in Fig. 2, modes 6 reducing to one single mode for the particular load sections chosen. However, mode 3 is not unlike a linear combination of modes 1 and 2, so that the cross inertia between these modes is high. This led to some difficulty on the flutter simulator[13] and mode 3 was therefore omitted for the quinary calculations. Since the mode can be represented fairly closely by modes 1 and 2, this does not necessarily mean any great loss in accuracy in the flutter solution, but it becomes more difficult to distinguish the types of flutter involving mode 3 from those involving modes 1 and 2.

3.2.2.1. *Outboard Mass at the wing tip*

The results for the mass on the leading edge are shown in Fig. 7. Fig. 7(*a*) shows the results for large roll inertia of the central mass. The flutter is initially of type (*a*) and a transition to type (*d*) occurs when the tip mass is about 0·18 of the wing mass. The asymptotic value for type (*d*) flutter is defined by the binary calculation using modes 4 and 6. The ternary solution using modes 1, 2 and 5 (Fig. 2) defines the region for type (*a*) flutter but shows an ultimate transition to type (*e*) flutter rather than to type (*d*). All five modes are necessary to obtain the correct solution. The flutter curve is similar to that for the fixed root wing (Fig. 4(*a*)).

Figure 7(*b*) shows the results for zero roll inertia of the central mass. Here there is first a transition from type (*e*) to type (*f*) flutter when the mass ratio is about 0·2, followed by a further transition to type (*h*) flutter when the mass ratio is about 0·46. The asymptote for type (*h*) flutter is provided by the binary solution using modes 3 and 6 (Fig. 2). The ternary solution using modes 1, 2 and 5 predicts the type (*e*) flutter region closely and also the transition to type (*f*) flutter, but does not show the type (*h*) branch. However the whole curve is reproduced to within about ± 3 per cent by a quaternary calculation omitting mode 6.

The results for the mass on the flexural axis are shown in Fig. 8. With large roll inertia of the central mass (Fig. 8(*a*)) the flutter is initially of type (*a*) with a transition to type (*b*) when the mass ratio is about 0·67. This is almost twice the mass ratio required for transition in the fixed root case (Fig. 4(*b*)). The type (*a*) flutter region is predicted closely by the modes 1, 2, 5 ternary solution and the asymptote for type (*b*) flutter is provided by the modes 4, 5 binary solution. Further, the quaternary omitting mode 6 is everywhere within ± 3 per cent of the quinary solution.

With zero roll inertia of the central mass (Fig. 8(*b*)) the flutter is initially of type (*e*) with a transition to type (*f*) when the mass ratio is about 0·85. The 1, 2, 5 ternary solution predicts the type (*e*) flutter region (but rather surprisingly does not predict the type (*f*) region) and the 3, 5 binary provides the asymptote for the type (*f*) flutter. The quaternary omitting mode 6 is everywhere within ± 3 per cent of the quinary solution.

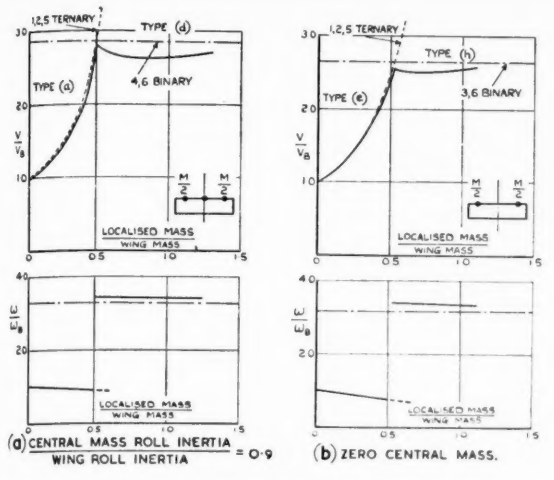

FIGURE 9 (*a* and *b*). Roll freedom—central mass—outboard mass on leading edge at two-thirds span.
$V_B = 538$ ft./sec.
$\omega_B = 236$ rads./sec.

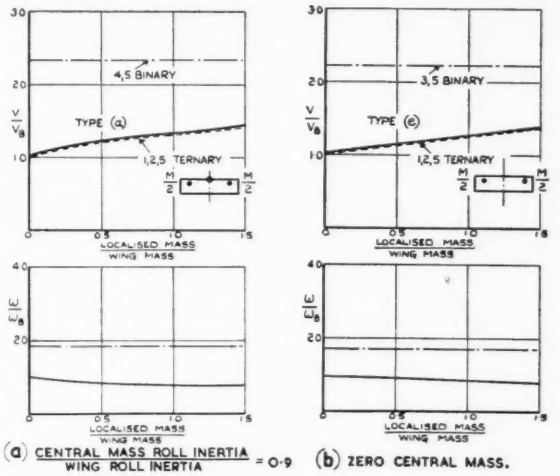

FIGURE 10. (*a* and *b*). Roll freedom—central mass—outboard mass on flexural axis at two-thirds semi-span.
$V_B = 538$ ft./sec.
$\omega_B = 236$ rads./sec.

3.2.2.2. *Outboard Mass at Two-thirds Semi-span*

The results for the mass at the leading edge are shown in Fig. 9. With large roll inertia of the central mass (Fig. 9(*a*)) the flutter is initially of type (*a*) with a transition to type (*d*) when the mass ratio is about 0·5. The modes 1, 2, 5 ternary solution predicts the region for type (*a*) flutter and the 4, 6 binary provides the asymptote for type (*d*) flutter. All five modes are required to reproduce the complete flutter curve, although it is closely similar to that for the fixed root wing (Fig. 5(*a*)) indicating that the roll freedom is not very important in this case.

With zero roll inertia of the central mass a transition from type (*e*) to type (*h*) flutter occurs when the mass ratio is about 0·52. The 1, 2, 5 ternary provides the type (*e*) flutter region and the 3, 6 binary provides the asymptote for type (*h*) flutter. Further, the quaternary omitting modes 6 provides a flutter curve that is within ±3 per cent of the quinary solution throughout.

The results for the mass on the flexural axis are shown in Fig. 10. No flutter transitions occur with either large or zero roll inertia of the central mass for the range of variation of outboard mass considered. With large roll inertia of the central mass the flutter is type (*a*) throughout, and with zero roll inertia of the central mass it is type (*e*) throughout. In both cases the whole flutter region is predicted closely by the modes 1, 2, 5 ternary solution.

3.3. PITCH AND TRANSLATION FREEDOM

To avoid longitudinal instability the centre of gravity of the wing must be ahead of the aerodynamic centre of pressure (at the quarter chord). To achieve this a concentrated central mass that is 0·84 of the wing mass is located at the wing leading edge for all the cases considered.

3.3.1. *Central Mass, no Outboard Masses*

The modes used in the calculation for this case are translation, fixed root bending, pitch and fixed root torsion, *i.e.* modes 1, 2, 5 and 6 of Fig. 3. The effect of variation in the pitching moment of inertia of the central mass is investigated.

The results are shown in Fig. 11. The flutter is initially of type (*k*), a transition to type (*a*) flutter occurring when the ratio of the pitch inertia of the central mass to that of the wing is about 3·2. The asymptote for type (*a*) flutter is provided by the modes 2, 6 binary solution, but all four modes are required for the complete flutter curve.

3.3.2. *Central Mass and Outboard Mass*

The cases are considered of an outboard mass at the wing tip and at two-thirds semi-span, with chordwise mass position on the leading edge and on the flexural axis, and with both large and zero pitch inertia of the central mass. These two central mass conditions ensured that either type (*a*) or type (*k*) flutter could be obtained with small outboard mass value.

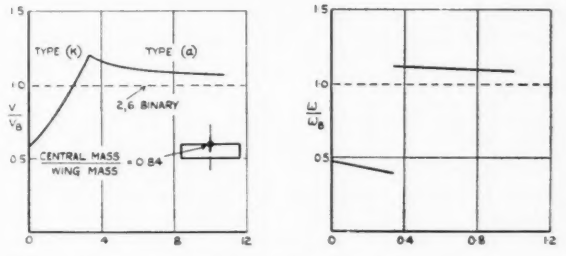

FIGURE 11. Pitch and translation freedom—central mass—no outboard masses.
$V_B = 538$ ft./sec.
$\omega_B = 236$ rads./sec.

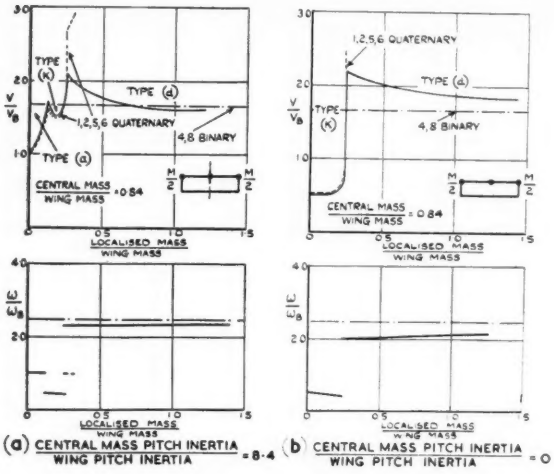
TYPE (k)
TYPE (d)
1,2,5,6 QUATERNARY
4,8 BINARY
TYPE (a)
CENTRAL MASS / WING MASS = 0·84
LOCALISED MASS / WING MASS
(a) CENTRAL MASS PITCH INERTIA / WING PITCH INERTIA = 8·4
(b) CENTRAL MASS PITCH INERTIA / WING PITCH INERTIA = 0

FIGURE 12 (*a* and *b*). Pitch and translation freedom—central mass—outboard mass on leading edge at tip.
$V_B = 538$ ft./sec.
$\omega_B = 236$ rads./sec.

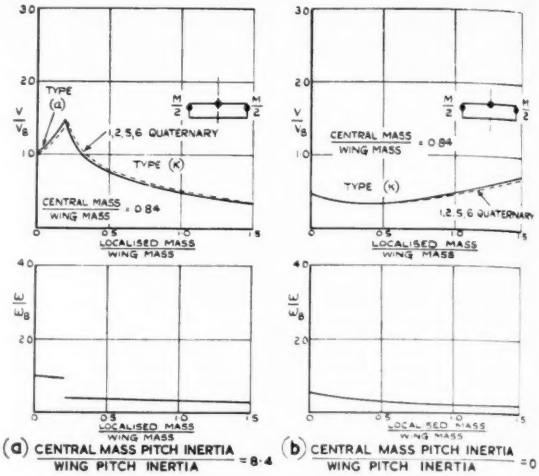
TYPE (a)
1,2,5,6 QUATERNARY
TYPE (k)
CENTRAL MASS / WING MASS = 0·84
LOCALISED MASS / WING MASS
(a) CENTRAL MASS PITCH INERTIA / WING PITCH INERTIA = 8·4
(b) CENTRAL MASS PITCH INERTIA / WING PITCH INERTIA = 0

FIGURE 13 (*a* and *b*). Pitch and translation freedom—central mass—outboard mass on flexural axis at tip.
$V_B = 538$ ft./sec.
$\omega_B = 236$ rads./sec.

The modes required in the calculations in these cases are shown in Fig. 3. With the mass at the tip only eight modes are necessary since the two part modes 7 and 8 reduce to single modes. With the mass at two-thirds span, modes 8 reduce to one single mode but modes 7 remain as two parts. However, in both cases the number of modes ideally required is beyond the capacity of the R.A.E. Simulator[13] and only six modes can be used. The modes omitted are modes 3 and 7. An approximation to mode 3 can be obtained by linear combinations of modes 1, 2 and 4, and modes 7 by linear combinations of modes 1, 6 and 8, but it becomes more difficult to distinguish the different types of flutter.

3.3.2.1. *Outboard Mass at the wing tip*

The results for the mass on the leading edge are shown in Fig. 12. Fig. 12(*a*) shows the results for large pitch inertia of the central mass. The flutter is initially of type (*a*) with a transition to type (*k*) when the ratio of tip mass to wing mass is about 0·12, followed by a further transition to type (*d*) when the mass ratio is about 0·25. The type (*a*) and type (*k*) regions are predicted closely by a quaternary calculation using modes 1, 2, 5 and 6 and the asymptote for type (*d*) flutter is provided by the binary using modes 4 and 8. All six modes are required for the complete flutter curve.

When the pitch inertia of the central mass is zero (Figs. 12(*b*)) the flutter is initially of type (*k*) with a very abrupt transition to type (*d*) when the mass ratio is about 0·24. The modes 1, 2, 5, 6 quaternary calculation provides the type (*k*) flutter region, the 4, 8 binary provides the asymptote for type (*d*) flutter, and all six modes are required for the complete flutter curve.

The results for the mass on the flexural axis are shown in Fig. 13. With large pitch inertia of the central mass (Fig. 13(*a*)) the flutter is initially of type (*a*) with a transition to type (*k*) when the mass ratio is about 0·2. For the latter type of flutter very low flutter speeds are obtained with large mass values. The whole flutter region is predicted closely by the modes 1, 2, 5, 6 quaternary calculation.

When the pitch inertia of the central mass is zero (Fig. 13(*b*)) the flutter is of type (*k*) for the whole range of mass variation considered, and very low flutter speeds are obtained. The flutter curve is predicted closely by the modes 1, 2, 5, 6 quaternary calculation.

3.3.2.2. *Outboard Mass at Two-thirds Semi-span*

The results for the mass at the leading edge are shown in Fig. 14. With large pitch inertia of the central mass (Fig. 14(*a*)) a transition from type (*a*) to type (*d*) flutter occurs when the mass ratio is about 0·48. The modes 1, 2, 5, 6 quaternary calculation predicts the type (*a*) region closely and the modes 4, 8 binary provides the asymptote for type (*d*) flutter. All six modes are required for the complete curve, but the curve is very similar to that for the fixed root wing (Fig. 5(*a*)).

When the pitch inertia of the central mass is zero (Fig. 14(*b*)) there is a transition from type (*k*) to type (*d*) flutter when the mass ratio is about 0·53. The modes 1, 2, 5, 6 quaternary calculation predicts the type (*k*) region closely and the 4, 8 binary provides the asymptote for type (*d*) flutter. All six modes are required for the complete curve.

The results for the mass on the flexural axis are shown in Fig. 15. With large pitch inertia of the central mass (Fig. 15(*a*)) there is a transition from type (*a*) to type (*m*) flutter when the mass ratio is about 0·4. For the latter type of flutter the flutter speed falls to a low value as the localised mass is increased. Calculations using the modes 1, 2, 5, 6 predict the type (*a*) flutter region closely and the whole flutter curve is obtained to within ± 3 per cent from a calculation using the five modes 1, 2, 4, 5, and 6.

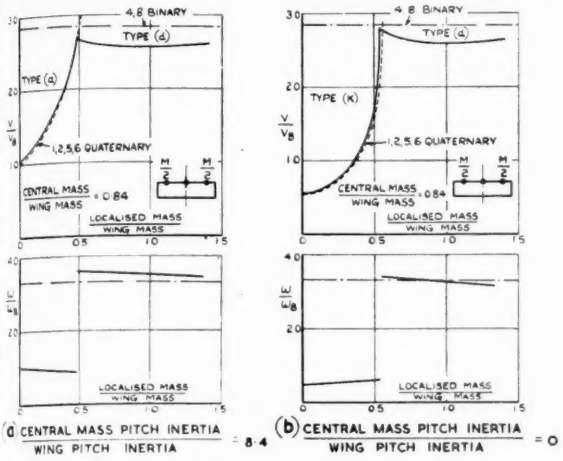
4,8 BINARY
TYPE (d)
TYPE (a)
TYPE (k)
1,2,5,6 QUATERNARY
CENTRAL MASS / WING MASS = 0·84
LOCALISED MASS / WING MASS
(a) CENTRAL MASS PITCH INERTIA / WING PITCH INERTIA = 8·4
(b) CENTRAL MASS PITCH INERTIA / WING PITCH INERTIA = 0

FIGURE 14 (*a* and *b*). Pitch and translation freedom—central mass—outboard mass on leading edge at two-thirds semi-span.
$V_B = 538$ ft./sec.
$\omega_B = 236$ rads./sec.

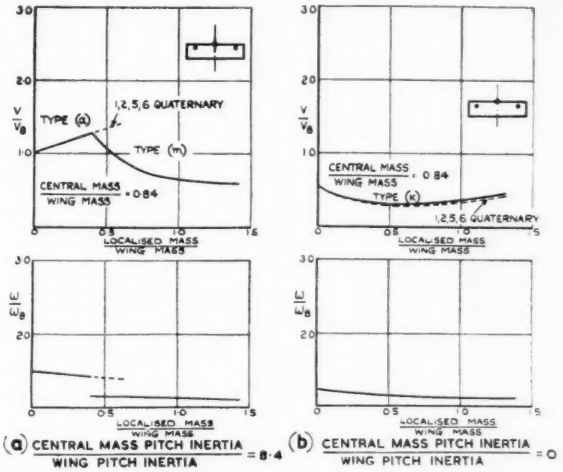
TYPE (a)
TYPE (m)
1,2,5,6 QUATERNARY
CENTRAL MASS / WING MASS = 0·84
TYPE (k)
LOCALISED MASS / WING MASS
(a) CENTRAL MASS PITCH INERTIA / WING PITCH INERTIA = 8·4
(b) CENTRAL MASS PITCH INERTIA / WING PITCH INERTIA = 0

FIGURE 15 (*a* and *b*). Pitch and translation freedom—central mass—outboard mass on flexural axis at two-thirds semi-span.
$V_B = 538$ ft./sec.
$\omega_B = 236$ rads./sec.

With zero pitch inertia of the central mass the flutter is of type (*k*) throughout, and low flutter speeds are obtained. The flutter region is predicted closely by the modes 1, 2, 5, 6 quaternary calculation.

3.4. DISCUSSION

Despite the limited nature of this theoretical investigation some important facts emerge, confirming earlier deductions. These are:—

1. Although in theory there would appear to be a large number of possible types of flutter for wings with localised masses, in practice only a few of these types are obtained. For the arrangements considered here 20 different types of flutter are possible theoretically, but only 8 are obtained.

2. An aircraft initially free from a type of flutter involving body motions may develop this type of flutter when carrying localised masses. This is evidenced by Figs. 12(*a*), 13(*a*) and 15(*a*).

3. With forward positions of localised mass a transition ultimately occurs to a type of flutter that is comparatively insensitive to increase in the localised mass, provided that the mass is large enough. (Figs. 4(*a*), 5(*a*), 7, 9, 11, 12, 14).

The flutter speeds associated with these types of flutter are generally higher than those for the bare wing. Their values vary with spanwise position of the mass and, for the cases considered here, are highest when the mass is at two-thirds semi-span (compare Figs. 4(*a*), 7, 11, 12 with Figs. 5(*a*), 9, 14). The flutter speeds with a large mass at two-thirds semi-span and forward of the flexural axis are, on the average, about 2·6 times that for the fixed root bare wing, as compared with about 1·6 for the mass at the tip.

4. When the mass is on the flexural axis, transitions to other forms of flutter may or may not occur for the range of mass considered here, depending on the spanwise position of the mass and the rigid body freedoms of the wing.

When the wing is either fixed at the root, or has roll freedom, the flutter speeds are generally higher than for the fixed root bare wing (Figs. 4(*a*), 5(*b*), 8, 10), but when there is freedom in pitch and vertical translation, flutter speeds far lower than that of the fixed root bare wing may be obtained (Figs. 13, 15).

4. Conclusions

From a general consideration of the available data on the flutter of wings with localised masses certain deductions are made as to the possible types of flutter that can occur. On the basis of these deductions it is shown that there is an optimum choice of modes for use in flutter calculations for wings with localised masses. These modes are obtained with artificial constraints imposed on the wing at the localised mass section fixing the wing at this section in translation and/or pitch.

It is deduced that for certain mass locations, types of flutter are obtained that are insensitive to increase of localised mass, with flutter speeds considerably greater than that of the fixed root bare wing. It is also deduced that for the majority of aircraft configurations the maximum flutter speeds for these types of flutter will be realised when the localised mass is in the region of two-thirds semi-span from the root.

A limited theoretical investigation is made for a rectangular unswept uniform wing with symmetric and antisymmetric body freedoms, which shows that with a heavy localised mass on the wing leading edge at two-thirds semi-span flutter speeds, some 2·6 times that for the fixed root bare wing are obtained.

It would appear that a judiciously placed localised mass can provide a convenient means of increasing wing flutter speed and, may be more economic from the weight viewpoint, than attempting to obtain a

corresponding increase by increasing wing skin thickness. At the same time the investigation shows that an ill-placed localised mass can reduce the wing flutter speed to a very low value.

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Design for Production

by

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1. Introduction

The title of the paper infers that, in general, aircraft design is not fundamentally suitable for production and, for various reasons, design for quick and easy production must be the result of compromise. In practice optimum design and easy production never seem to coincide exactly. Refinement of design for production covers many broad conceptions.

In its broadest sense it should be the uppermost consideration from the very beginning of the design of the prototypes. The primary object of designing any new aircraft, civil or military, is to be able to produce a series of the most efficient fully approved and tested production aircraft at the highest possible rate in the shortest possible time. The first part of the paper, therefore, deals with the design and production planning phases of a new aircraft when, by joint consultation with the production departments, the design is viewed from the production angle from the outset. One of the most important considerations here is the breaking down of the aircraft into components of minimum economical size to reduce "in jig" time which is the most important factor in the manufacturing time cycle. The compromise between the added weight for additional joints, compared with the saving in production costs, must be carefully considered. It is also necessary to ensure that structural specimens are made available at the right time and that all ground test work is completed at the proper time, to avoid flight restrictions.

In its narrowest sense the revision of detail design to allow fabrication by more economical methods of production offers a wide scope for economies in time and cost and in between these two extreme interpretations of the title of the paper, pros and cons of various basic types of construction offer fruitful fields for discussion.

2. Planning for Production

2.1. ESTIMATION OF DESIGN AND PRODUCTION TIMES

The foundation on which the grand plan for instituting a production run is based must be the promised dates by which the drawings will be issued. By reference to records of previous aircraft with suitable adjustments for size factor, complication factor, operational role, and so on, estimates must be made of target dates for the issue of sufficient drawings to produce the first prototype. As a matter of interest, the author's firm is now using the Hollerith system of recording and analysing D.O. records. In Fig. 1 the horizontal lines give the period over which drawings will be issued, the left hand ends giving the dates of issue of the first drawing and the right hand ends the dates on which sufficient drawings are available to build the first prototype component. The top two lines also have marked on them the percentage of the total number of drawings to be issued by the given dates, so that by use of a normal type of progress sheet the actual achievements can be continuously compared with the estimates.

Correct phasing of these components is essential if valuable time is not to be wasted. There has been too much of a tendency in the past to tackle the apparently easy things first. Thus, tailplanes, fin, rudder, elevators, flaps and ailerons, because of their relative simplicity have been drawn out and issued as quickly as possible, thus apparently impressing the production departments with how quickly the design office is getting on with the job. The shops, in their turn, also fall for this fatally easy exercise and the result is a collection of completed components getting dusty in the stores long before the real work on the fuselage and wings has been properly started. Wind tunnel results then begin to flow into the Design Office, indicating changes to these completed components, engendering bad feeling between Works and Design and resulting in a waste of time and money.

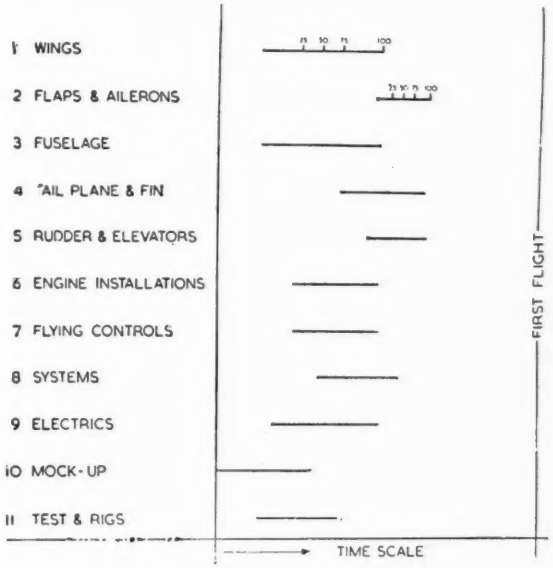

FIGURE 1. Issue of drawings for first prototype.

A Section Lecture given on 29th January 1957.

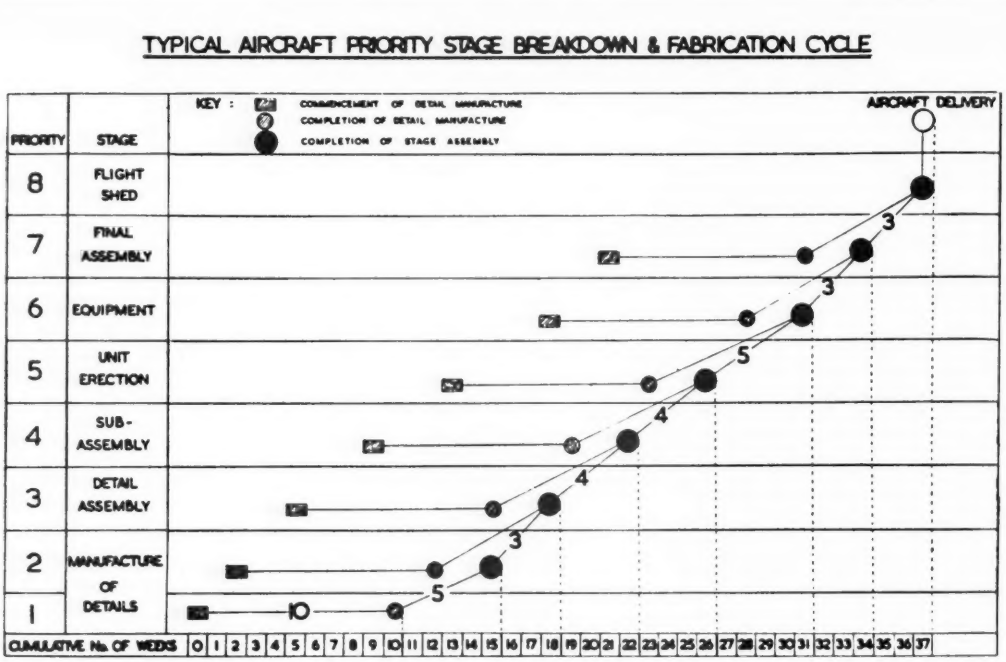
TYPICAL AIRCRAFT PRIORITY STAGE BREAKDOWN & FABRICATION CYCLE
KEY :
COMMENCEMENT OF DETAIL MANUFACTURE
COMPLETION OF DETAIL MANUFACTURE
COMPLETION OF STAGE ASSEMBLY
AIRCRAFT DELIVERY
PRIORITY
STAGE
8 FLIGHT SHED
7 FINAL ASSEMBLY
6 EQUIPMENT
5 UNIT ERECTION
4 SUB-ASSEMBLY
3 DETAIL ASSEMBLY
2 MANUFACTURE OF DETAILS
1
CUMULATIVE No. OF WEEKS 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37

FIGURE 2. Programme of manufacturing "Priorities."

How much better to start on the difficult production tasks first; the large components which require more complicated jigs and tools, more floor space and, what is far more important, contain more equipment, always the bottle neck in any aircraft production line. Thus, in the author's view, the smaller components should be left until later when there is less likelihood of alteration and to allow a proper emphasis by the production departments on the difficult components first. Meanwhile the production departments have been working on similar lines and have produced an estimated aircraft delivery programme. A specimen of this is shown in Fig. 2 and this indicates the flow of details and subassemblies to final assembly and first flight.

Each part or assembly is given a priority to indicate its position in the fabrication cycle. The priority of any part is determined by its proximity or remoteness to the completed aircraft. Normally this is the next lower priority to the assembly in which it is fitted, but for parts with an exceptionally long "make" cycle time, *i.e.* longer than 10 weeks, a still lower priority number may be allocated. Although the early priorities (1 and 2) are mainly Machine Shop and Press Shop parts, and main assemblies carry the later priorities (4, 5 and 6), there are many small parts which have later priorities if they are needed at a later stage of assembly. Each part or assembly has one priority only, although it may be used at two or more stages in fabrication. In these cases the priority is determined by the earliest requirement.

In the example, the eight priorities or stages are spread over a period of 37 weeks, the period between the individual priorities are determined by the type of work involved, *e.g.* the Equipment stage demands a period of five weeks. The number of weeks allowed is sufficient for the assembly requiring the longest period of erection, including paint requirements. Within limits the periods between priorities can be varied to facilitate changes in scheduled rates of output.

An assembly draws its constituent parts from both the assembly and "making" in the previous priority. In the case of "making," completion is called for three weeks before the issue of groups to allow for inspection, treatment, and so on.

A longer fabrication period is needed when a new aircraft is being brought into production. This extension can be kept to a minimum by issuing small initial batches of work.

From the stage breakdown the following programmes can be devised:—

Material delivery dates.
Tool completion dates.
Works order issue dates.
Group issue dates to shops.
Batch completion dates.
Line assembly requirements.

The estimated flight date can only be determined when the drawing issue programme and the production programme are welded together in a logical sequence.

2.2. INTEGRATION OF TEST SPECIMENS WITH PROTOTYPE COMPONENTS

The days when the sole object of the workshop organisation was to concentrate manufacture on those parts required for the first prototype in order to get a shiny new aircraft which will just about fly on the earliest possible date (and then claim a new record) are now, fortunately, disappearing. Manufacturers as a whole, and production teams in particular, are beginning to realise that a prototype aircraft without a pressure sealed structure or air conditioning equipment,

restricted in its speed range by non-completion of structural tests, resonance tests and the like, is more of a liability than an asset. Further, this contraption then goes to the S.B.A.C. Show where it obviously doesn't fly as fast as it should and, because of its early appearance, emphasises in the public mind the very long time between this and the time when production aircraft are in service. These days large structural test specimens amount to two additional (unequipped) aircraft and the production departments must be prepared to allocate these extra components very early in the programme if the full benefit is to be obtained from the prototype aircraft.

Figure 3 is an allocation chart covering the first five sets of major components, two of which are for test and three for prototypes. The first fuselage has been allocated for static pressure tests and is also regarded by the workshops as a useful exercise in establishing sealing techniques in preparation for the fully operational first prototype. In parallel with these arrangements test rigs have to be prepared to match up with the delivery dates of the specimens and here the entire resources of the company should be available to see that these programmes too, are met.

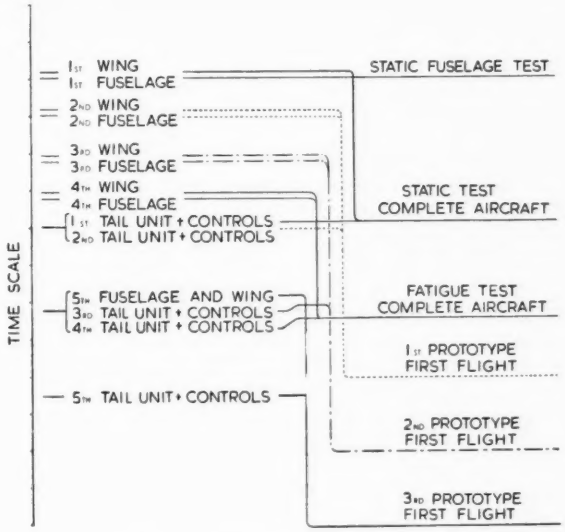

FIGURE 3. Allocation of first five airframes.

2.3. INTEGRATION OF ENGINEERING PROGRAMME WITH PRODUCTION

Mention has already been made of the necessity of completing structural test specimens before flight and of the early completion of wind tunnel tests before the construction of control surfaces and stabilisers. There are many other facets to the successful initiation of a fully approved production programme and the whole picture from initial design to final product is illustrated in the phasing diagram shown in Fig. 4. This shows the target for completion of the various stages with two main overall objectives. Firstly, to construct prototype aircraft with as few flying and other operational restrictions as possible. Secondly, to establish a production run of fully approved and tested aircraft.

The chart in Fig 4 can be used as a Master Progress Chart to assess achievement against targets and to highlight the necessity for making special efforts to correct any items which may become out of phase. Each item should have its own separate planning chart for this purpose. The diagram illustrates many important principles if the project is to be successful:—

(*a*) Wind tunnel tests should be completed as soon as possible so that stabilisers, control surfaces, and so on may be finalised in time to avoid changes in the shops.

(*b*) Mechanical tests on detail parts should also receive top priority in design and manufacture for the same reasons as given in (*a*).

(*c*) As much engineering development on all systems such as hydraulics, cabin air conditioning, fuel systems, engine installation, and so on, should be completed on ground rigs before these items are installed in the aircraft.

(*d*) The static tests on the airframe should be completed before the flight of the first prototype to avoid strength restrictions on flight manoeuvres or pressurisation.

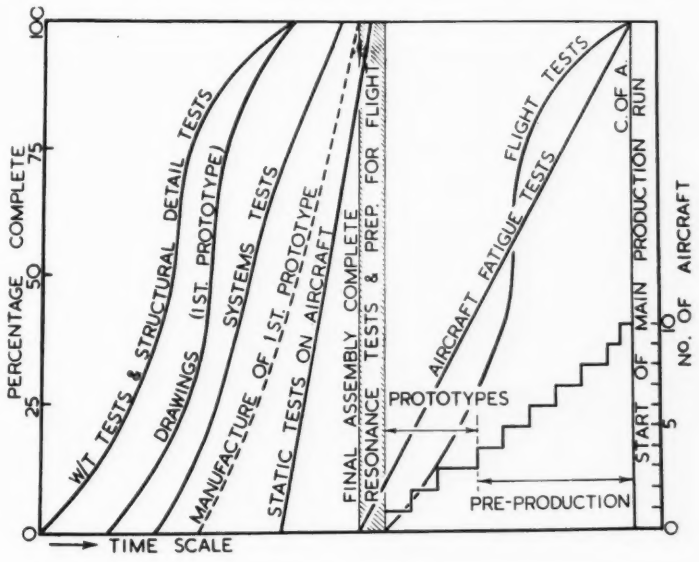

FIGURE 4. Engineering and production phasing diagram.

(*e*) Stiffness and resonance tests should be completed and assessed before the flight of the first prototype to avoid speed restrictions during early flight tests.

(*f*) Fatigue tests, both detail and airframe, should be completed by the beginning of the production programme.

(*g*) Flight tests, including winterisation and tropical trials, should be completed before the production run starts.

(*h*) C.A. Release or C. of A. should be obtained before delivery of first production aircraft.

This may seem an over-idealised picture of the prerequisites to early production. It cannot be denied, however, that all these enterprises need to be successfully completed. The author believes that all departments of the organisation, including production, should be fully alive to the overall picture.

3. Design Office Equipment

There is no doubt that the quality and type of equipment, and the conditions under which they are used can have a marked influence on output in design.

The British Specialist Team who visited the U.S.A. under the auspices of the Anglo-American Council on Productivity reported that companies had paid special attention to drawing equipment and adequate accommodation and amenities, even to the extent of experimenting with multi-colour schemes of decoration. One firm reported a "considerable" increase in output with the same staff, merely by moving to more commodious accommodation. Adequate reference space is a prerequisite for any designer aiming at productivity. The loss of valuable time by design draughtsmen trying to make do with a very limited reference table is considerable. A comfortable, adjustable seat with a back and a readily adjustable drawing table with a light green backing paper, allied to a shadowless lighting system giving not less than 75 foot candles at board level is suggested as an ideal. Drawing machines of various types to suit office preference are available, but the old Tee square has undoubtedly outlived its usefulness as a production instrument.

Figure 5 shows the Aero Design Office at the new A.W.A. Research Centre at Whitley and Fig. 6 the type of draughting equipment which is being installed to replace the old drawing tables.

4. Production Methods

4.1. ESTABLISHED METHODS

It is essential that the Design Office be fully aware of the facilities available immediately in the workshops. An enterprising production organisation is constantly adding new equipment and introducing new processes which are potential improvements in production times.

FIGURE 6. Draughting Equipment.

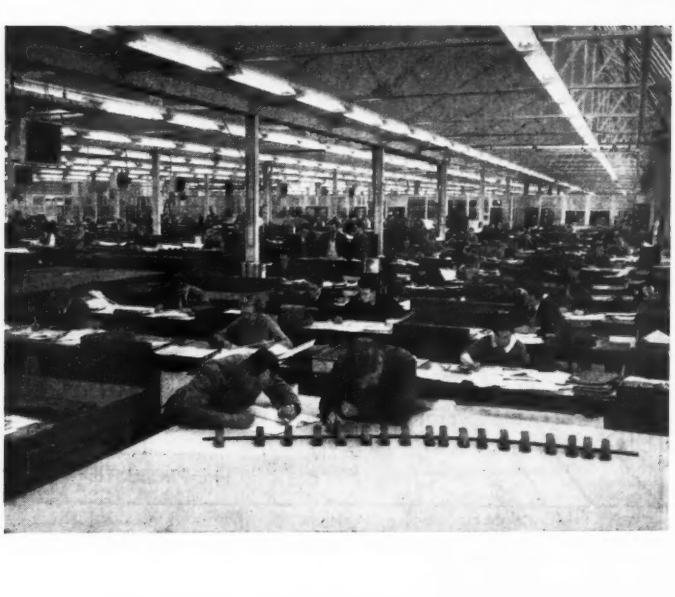

FIGURE 5. View of A.W.A. Drawing Office at Whitley

Fig. 7 depicts a page from a manual giving details of all the "special plant" installed in the author's works. Copies of this handbook are circulated to all Section Leaders and filed for reference in the Technical Library.

4.2. TYPES OF CONSTRUCTION AND NEW METHODS

The type of construction necessary for a new project may be dictated by design considerations if, for instance, flight speeds are much advanced beyond current designs. Early consultation with works departments is necessary to obtain new plant and introduce new techniques. In less sophisticated regimes, such as that of current civil aircraft, alternative methods are possible and here a compromise must be made between what is essential for design and what is expedient in the works. This process can also work in reverse where the Production Departments claim production advantages for new methods which have to be examined for suitability by the Design Office. In general it is difficult to introduce entirely new methods into a new design without causing delay and this is where a constant review of new methods during times when no new aircraft are being produced is essential. It is difficult to forecast what will be needed in the future, however, unless a particular organisation concentrates on one type only and is in a strong position to anticipate the next step. For those less fortunate who have no idea what their next major project might be it is practically impossible to anticipate future needs.

Of new forms of construction, integral machining, metal gluing, metal honeycomb construction and chemical milling can be mentioned as all requiring a considerable amount of special plant preceded by considerable research programmes, the absence of which at the start of a new project can considerably lengthen the gestation period. If, for instance, metal gluing of parts in locations particularly liable to fatigue is considered essential by the Design Office, early notice must be given to the Production Departments to enable them to obtain the necessary plant.

5. Liaison Between Production and Design Departments

The key to success of guiding a new project through all its stages as previously described is effective liaison between production and design at all levels. Day to day queries from the shop floor are usually handled through the process planning department representatives using shop query forms with or without prior verbal consultation with the particular Design Office Section concerned. This scheme, to work well, demands immediate and sympathetic consideration by the Design Office and depends for its success on the goodwill existing between the departments. Modern tendencies for Design Offices and Works to be separated makes this harder to work and intrinsically demands a higher standard of accuracy and clarity on initial drawings if irritating delays are not to occur. In the design stage many methods of liaison are current. Two systems only will be dealt with here, *i.e.* Production Advisors and the Development Committee.

BRAKE PRESSES

Type	Besco			
Maker's name	Edwards	Verson	Edwards	Craig & Donald
Distance between housings	71 in.	79 in.	12 ft. 7 in.	12 ft. 2 in.
Length of bed and ram	81 in.	96 in.	14 ft. 6 in.	13 ft. 0 in.
Rated bending capacity	20 tons	75 tons	100 tons	100 tons
Die opening	10 in.	13¾ in.	13½ in.	12¼ in.
Standard stroke	2 1/16 in.	3 in.	3 in.	3 in.
Adjustment of ram	3 in.	5 in.	3 in.	5 in.
Strokes per minute	28 in.	36 in.	24 in.	30 in.
Ram plate thickness	2 in.	2½ in.	2½ in.	3 in.
Housing plate thickness	4½ in.	2 in.	2½ in.	3 in.
Number of machines available	1	1	1	1

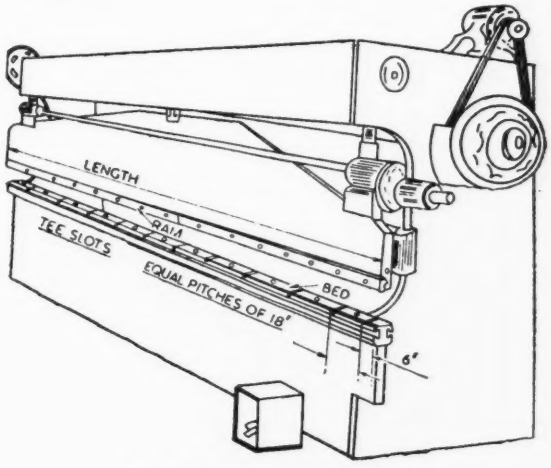

FIGURE 7. Page from plant handbook.

Production Advisors are usually people with a wide experience of production methods current in the factory and who are attached to the Design Office. Their duties vary in different organisations from merely consultative to a responsibility for vetting all drawings for suitability for production.

The Development Committee, to be successful, must consist of members of very high executive standing with the power to make decisions on the spot, or to order more detailed investigations by sub-committee. This committee should meet as soon as the Chief Designer can give any indication of the probable shape and size of the project and certainly well in advance of issue of drawings. The regular members may consist of the following:—

Production Departments:
Works manager.
Chief planning engineer.
Assistant production manager in charge of project.
Members of general manager's staff.
Quality manager.
Process development manager.
Chief estimator.
Chief superintendent.
Chief jig and tool designer.

Design Department:
Chief designer.
Asst. chief designer (projects).
Asst. chief designer (technical).
Chief project designer.
Chief structural and mechanical engineer.

Other members can be co-opted when necessary for special problems which are their chief concern such as:—

Chief test pilot.
Weights engineer.
Transport manager.
Chief buyer.
Chief mechanical test engineer.
Flight shed superintendent and other production personnel.

All aspects of the design and production should be covered by this committee which can be aptly described as the "steering committee" for the project. The Chief Designer or the Works Manager, as convenient, should act as chairman.

The work of this committee can be divided into three distinct phases:—

(*a*) To work out the best compromises between design requirements and introduction of new processes.
(*b*) General responsibility for watching progress and ironing out difficulties.
(*c*) To guide the smooth transition from prototypes to full production.

Two fundamental subjects which have been discussed recently by the Development Committee and special sub-committees at the author's works are referred to in Sections 6 and 7.

6. Drawing Systems

This is a subject which is constantly coming up for review. It is doubtful whether any ideal system really exists and, in any case, the system is often geared to the particular organisation in which it is to operate, causing quite a few headaches when parts are sub-contracted. The S.B.A.C. system has, however, gone a long way to solve this problem and most firms are now using it in one form or another. It is not proposed to enter into a detailed description of drawing systems here but one or two major points are worthy of note. Latest thoughts indicate that the lettering and numbering system should enable the location of a detail part to be recognised, *i.e.* the aircraft and component on which it is used. The great difficulty which arises here, of course, is the identification of standard parts. Clearly these must bear one identity only and, therefore, cannot conform to the ideal system. The character of drawings also varies considerably and there are quite large reductions in drawing office time possible by departing from the principle of one detail part, one drawing. Schedules on drawings, instead of separate schedule sheets, also leads to economy in drawing office time.

With regard to scheduling, further scope for economy exists since planning groups never seem to coincide with the drawing office schedule groups. Is it too much to hope that planning and drawing offices could be integrated to such an extent that only one set of schedules or groups need be written? Apart from duplication the present system can lead to many difficulties and misunderstandings, some quite unexpected. To give one example the weights department, to be able to do its job efficiently, must be acquainted with the planning groups in order to understand the weights supplied by the shops. Since target weights have been estimated from design grouping this leads to a considerable amount of difficulty.

Finally, a numbering system incorporating the fewest possible letters and digits results in a significant saving in clerical time when it is realised that one part number is likely to appear on design, planning, ordering, manufacturing, inspection, documents totalling at least 50 per part number (ignoring copies) and that something like 20,000 differently designed parts are needed to make an aeroplane.

7. Size of Units

This is the basis of planning. In general the smaller the unit size the better, giving reduction of "in-jig" times. This may demand the use of joints which might not otherwise have been necessary and design penalties from weight and fatigue must be carefully watched. The S.B.A.C. has issued an excellent pamphlet entitled "The Financial Aspect of Weight Saving" which is of great value in deciding just how far the design office can meet the shops. This general aspect has been referred to down the ages of aircraft design. The author can well remember Major F. M. Green's slogan in about the late 1920's—"Spend a pound (sterling) to save a

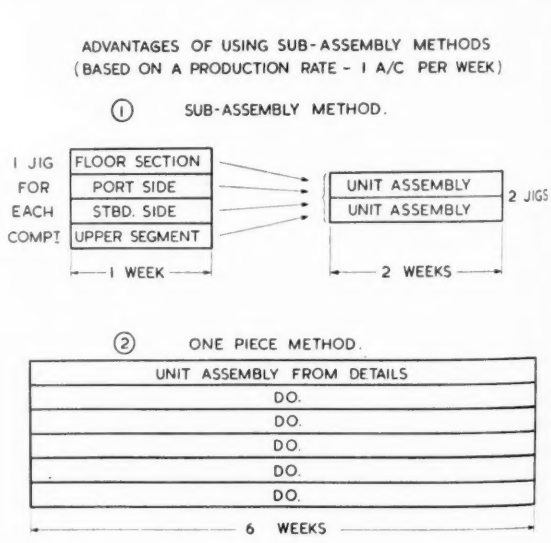

FIGURE 8. Effects of sub-assembly methods.

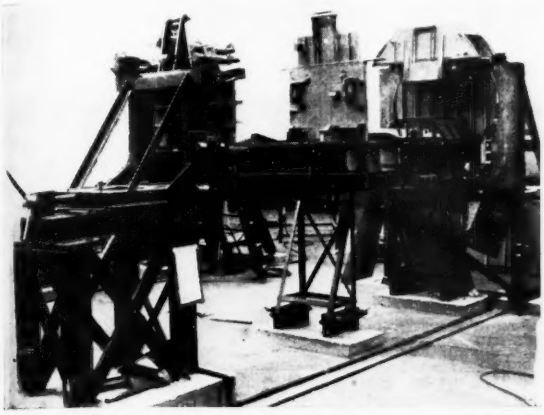

FIGURE 9. Fuselage floor and bulkheads.

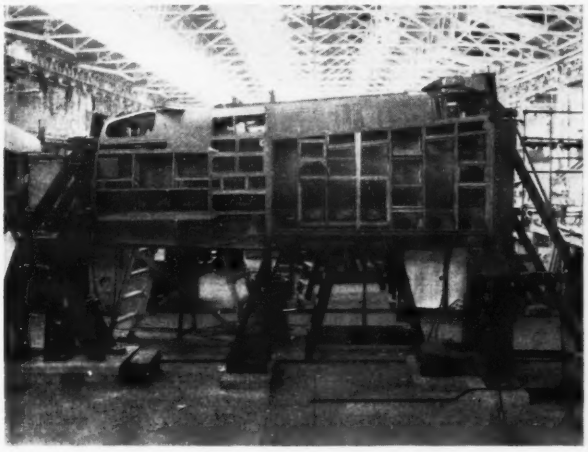

FIGURE 10. Fuselage assembly.

pound (weight)." Things have altered a lot since then and, depending on the type of operation, the value to a civil airline annual revenue of saving a pound (weight) is now more like £50. Assuming an eight year life it has to be demonstrated that a shops request to throw away one pound of structure weight to ease production should be accompanied by an assurance that the first cost of the aircraft will be reduced by £400. All joints need not, of course, be detachable; advantage can be taken of existing joints in sheets, stringers, and so on.

One of the fundamental requirements for efficient production is that the operation cycle times should be as short as possible. The shorter the cycle time the more efficient the operator(s) can become and similarly, the organisation surrounding the operator becomes more efficient. The operator has less to learn and in consequence ceases to refer to drawings, process instructions, and so on, much sooner than he would do with a longer cycle time and, furthermore, achieves a greater manual efficiency, thus reducing costs. Shop supervision is more efficient as it is dealing with several units of manageable proportions instead of one large unit often involving a rather lengthy and involved chain of operations, often inter-related. In general, the higher the production rate, the greater the amount of sub-assembly is required. Ideally, cycle times and jig times should be whole fractions of the shift time. Unfinished cycles at the end of a working day are highly inconvenient and particularly embarrassing in double shift working.

Figure 8 shows the effect of breaking a fuselage down into components. This shows that in order to achieve a production cycle of one per week either two main jigs and four (much smaller) sub-assembly jigs can be used instead of six much more complicated jigs to build up the structure complete. The latter method has several disadvantages which all add up to higher production costs. More floor area would be required. The man hours expended in productive work and supervisory work would also be greater and the learning period increased due to greater complication. Access would be greatly inferior, increasing time to get at awkward places inside the structure of the whole assembly which would be exposed in the sub-assemblies. Figs. 9 and 10 show how these principles were applied to a fighter fuselage.

Great advantages accrue from arranging joints in systems, controls, and so on, to correspond with unit sizes enabling each unit to be completed, equipped and tested before the final assembly stage. Finally, unit sizes have to be carefully watched, not only from the point of view of major repairs but they often have to be transported from one factory to another.

8. Refinement of Design to Improve Production

8.1. MACHINING LIMITS AND SHEET GAUGES

This thorny subject probably represents one of the most serious areas of conflict between the prime necessity of saving weight and ease of production. There can be no doubt that wider tolerances reduce costs of production whether it be in the machine shop or rolling mill. There can be no relenting here from the design office point of view and the shops must be urged to do the best they possibly can to keep to nominal sizes. The problem is very largely one of psychology and, recognising this, much thought has been given to statistical methods of strength assessment whereby the general shop performance is used instead of the most adverse drawing tolerances.

Consider a non-fitting part on which the shops usually demand $\pm 0{\cdot}010$ in. As the regulations stand at the moment it is mandatory to stress this part to $-0{\cdot}010$ on an outside diameter whereas in the majority of cases the shops produce the part to $+0{\cdot}010$. Assuming an average ruling dimension of one inch this represents an excess of weight of $4\frac{1}{2}$ per cent. Taking this as a basis and assuming that there are 5 per cent of such parts in the structure weight of an aeroplane a weight penalty of 0·2 per cent ensues or 0·07 per cent increase in take-off weight. Alternatively, this could result in a 0·2 per cent reduction in payload of a

FIGURE 11 (*left*). Wing tip.

FIGURE 14. Navigation light panel.

FIGURE 12. Stretch forming wing tips.

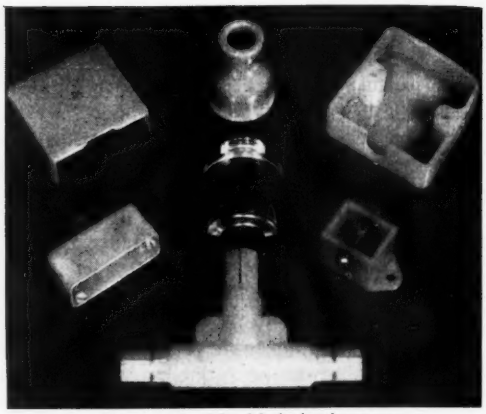

FIGURE 15. Moulded plastic parts.

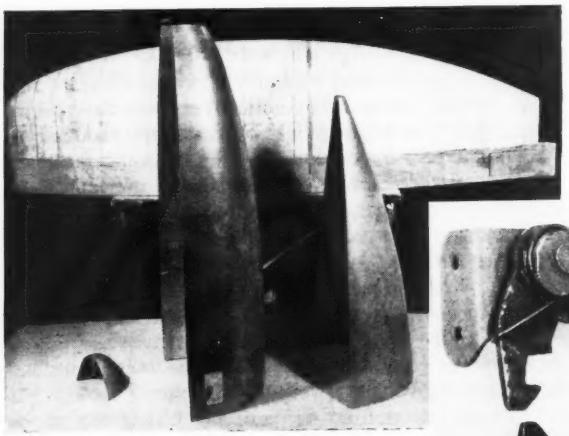

FIGURE 13. Stretch formed parts.

FIGURE 16 (*right*). Impact extrusions.

FIGURE 17 (above). Investment castings.

medium size civil aircraft reducing it by 50 lb. in 25,000 lb., causing an annual loss in revenue of £2,500. This is obviously a very serious matter and a great deal of attention has been paid to this subject by many authorities. One of the most notable papers was by Dr. A. E. Russell* which pointed the way to the use of close tolerance sheets now universally adopted by civil aircraft constructors. Potential saving in weight is even more spectacular in this instance amounting to 300 lb. on a 25,000 lb. payload, or equivalent to £15,000 per annum in revenue to the airlines. Much remains to be done, however, and a more vigorous pursuit of statistical methods is plainly indicated, plus an intensive educational campaign in the shops. Inspectors too must be urged to play their part and insist at least on the limits being worked to. There is evidence that so long as the part is over nominal size it has been accepted, however much in excess it may be.

8.2. "HANDING" OF PARTS

This is a very fruitful area for easing production costs. Jigs and tools can be significantly cut down if port and starboard parts can be made inter-changeable.

8.3. STRETCH FORMING

Figure 11 shows a wing tip which was originally made in soft aluminium in two halves on a drop hammer and finally welded together. It was later stretch formed on a Hufford machine, as shown in Fig. 12, enabling aluminium alloy to be used. The resulting job was not only cheaper and quicker but was more satisfactory from the strength and particularly, the handling point of view before assembly. Fig. 13 also shows some stretch formed parts originally designed in two halves with a butt strap down the middle, thus saving weight as well as cost and time. Fig. 14 shows a stretch formed part from which two (handed) navigation light panels were produced. Originally it was designed in two halves and produced by drop hammer. The stretch forming also eliminated the butt strap joint.

8.4. INJECTION MOULDING

Figure 15 shows a number of parts made by moulding plastics (nylon and polystyrene). This equipment has proved invaluable for producing special parts for aircraft and missile electrical systems which would otherwise have been machined from the solid.

8.5. IMPACT EXTRUSIONS

Figure 16 shows some examples of impact extrusions. On the left is a belled tube normally sealed by a welded plate. As an impact extrusion the part is in one piece, saving cost and resulting in a greatly superior article. The male and female sockets on the right have also been made by this method. In addition to the production advantages, improvements in grain flow result giving better fatigue properties. Normally such parts would be machined from bar.

8.6. INVESTMENT CASTINGS

Much has been written about the integration of structures and the advantages of reducing the number of separate detail parts, particularly for large components. Small components can often be "integrated" as illustrated in Fig. 17. The upper view shows the originally designed catch consisting of lever, catch and boss made from three parts and welded together.

The lower view shows the alternative version made in one piece by investment casting. This process produces close tolerance precision castings requiring no machining.

9. Conclusions

Design for production is a subject which covers a very wide field and can be treated in a variety of ways. This paper has tried to cover the subject broadly, covering the whole process of converting a new design into full production. It emphasises the importance of early joint planning between the two partners in the enterprise and suggests methods by which this may be done quickly and efficiently. The design office must know intimately the facilities of the workshops. The production departments must be told at the earliest possible stage of new processes essential to the success of the project. When production starts the design office must be willing to consider suggestions by the production departments for improvement. Above all a spirit of mutual co-operation must pervade the whole organisation if the desired success is to be achieved.

ACKNOWLEDGMENTS

The author wishes to thank Sir W. G. Armstrong Whitworth Aircraft Ltd., for permission to present the paper and his colleagues in both production and design departments for helpful suggestions and practical assistance in its preparation.

*Rationalisation of Aluminium Alloy Specifications, *Journal of the Royal Aeronautical Society*, January 1945, page 14.

Reinforced Elliptical Holes in Stressed Plates

by

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SUMMARY: This paper considers the problem of a reinforced elliptical hole in a plate under the action of a principal stress system of the type found in cylindrical and ellipsoidal pressure vessels. That is, stress systems in which the ratio of the principal stresses is not greater than two to one. It is shown that when the ratio of the major and minor axes of the ellipse can be chosen arbitrarily, practical reinforcements can be designed to give a maximum stress around the hole which is only slightly greater than the maximum stress in a similarly loaded plate with no hole. General expressions are obtained for the stress distribution in the plate around the hole, for the stress acting on a normal cross section of the reinforcement, and for the cross-sectional area of a reinforcement which gives a small stress concentration. These are used to find the variation in the stress distribution around the hole due to reinforcements having different cross-sectional areas when the applied principal stresses are in the ratio of two to one and Poisson's ratio for the material of the plate and reinforcement has practical values.

1. Introduction

The presence of a circular or elliptical hole in a flat plate loaded in its own plane results in an increase of the stress concentration. If the hole is small compared with the lateral dimensions of the plate its effect on the stress distribution will be local whereas a large hole may appreciably alter the stress distribution in the whole plate. The effect of suitably reinforcing the hole is to decrease this stress concentration so that when the stress in the plate is nowhere to exceed a given value the presence of the reinforcement results, except in the case of very large holes, in a decrease in the necessary weight of the structure. The design of a reinforcement is therefore an important problem when the weight of the structure must be as low as possible, as, for example, holes in the skin of an aircraft.

The solution for an unreinforced elliptical hole in an infinite plate under the action of applied principal stresses has been obtained by Inglis[1]. He found that when these stresses are equal the maximum stress occurs around the hole at the end of the major axis and is given by $2fa/b$, where f is the applied stress and a and b the semi-major and semi-minor axes of the ellipse, respectively. Furthermore, for a plate uniformly pulled in one direction the maximum stress is $f(1+2a/b)$ when the minor axis of the ellipse is parallel to the direction of the applied stress, and $f(1+2b/a)$ when the major axis of the ellipse is parallel to the direction of the applied stress.

Mansfield[2] considered a reinforced elliptical hole in a plate under the action of applied principal stresses, and derived expressions for the ratio of the major and minor axes of the ellipse and for the cross-sectional area of the reinforcement which makes the hole neutral.* He found that the required reinforcement had a variable cross-sectional area with maximum and minimum values at the ends of the major and minor axes, respectively, while the ratio of the major and minor axes of the required ellipse was proportional to $\sqrt{(f_1/f_2)}$, where f_1 and f_2 are the principal stresses with $f_1 > f_2$ and the major axis of the ellipse being in line with the direction of f_1.

For given principal stresses the difference between the maximum and minimum cross-sectional areas of the reinforcement depends on Poisson's ratio for the material, and for many practical purposes is sufficiently large to be prohibitive. For example, when the applied stresses are in the ratio of two to one, the ratio of the maximum and minimum cross-sectional areas of the reinforcement is 2·47 when Poisson's ratio is 0·25, and increases to infinity as Poisson's ratio is increased to 0·5.

From the design point of view, a practical reinforcement would have a constant cross-sectional area which could be chosen so that the stress concentration around the hole would be as low as possible. Thus, if the shape of the hole is of secondary importance, it is reasonable to suppose that practical reinforcements corresponding to small stress concentrations may be designed by making the hole as near neutral as possible, consistent with the condition that the cross-sectional area of the reinforcement is constant.

In this analysis it is shown that, when the reinforcement has a negligible bending stiffness in the plane of the plate and when there is a choice for the ratio of the major and minor axes of the ellipse, practical reinforcements can be designed so that the maximum stress around the hole is only slightly greater than the maximum stress in a similarly loaded unperforated plate, while the variation in the necessary cross-sectional area of the reinforcement is sufficiently small to be neglected. For example, when Poisson's ratio is 0·25 and the applied stresses are in the ratio of two to one, the maximum stress around the hole is 4·5 per cent greater than the maximum stress in a similarly loaded unperforated plate, the variation in the necessary cross-sectional area of the reinforcement is approximately 5 per cent, and the ratio of the major and minor axes of the ellipse slightly less than 1·6.

It should be noted that the results obtained here are

*Reinforced holes which do not affect the stress distribution in a plate are said to be "neutral."

Received 16th January 1957.

for a reinforced hole in an infinite flat plate under the action of stresses applied in its own plane. However, it can be shown that the stress concentration around the hole decreases rapidly along normal lines away from its boundary so that the solution can be applied to a finite plate provided that the least lateral dimension of the plate is several times greater than the major axis of the elliptical hole. Furthermore, it is reasonable to suppose that the solution is substantially correct for a reinforced hole in a curved plate or pressure vessel provided that the ratio of the radius of curvature of the plate to the width of the hole is reasonably large. The application of the solution to this case is equivalent to the assumption that the bending stresses induced in the plate around the hole have a negligible effect on the distribution of the membrane stresses.

NOTATION

A_R	cross-sectional area of reinforcement at any particular point
a	semi-major axis of the ellipse
b	semi-minor axis of the ellipse
c	constant
E	Young's modulus
F	force acting on a section of the reinforcement normal to the curve of the hole
f	applied stress
f_1, f_2	principal stresses
h	thickness of the plate
x, y	Cartesian co-ordinates
η	variable
θ	angle between the x-axis and a tangent to the curve forming the hole
ξ	variable
σ	Poisson's ratio
φ	function defined in equation (1)
ω	parameter

2. Procedure

Let the principal stresses acting on the boundary of the plate in the x- and y-directions be nf and f, respectively. This system of loading, which is shown in Fig. 1(*a*) and called state 1, can be resolved with the aid of a parameter ω into the stress systems shown in Figs. 1(*b*) and 1(*c*). That is, states 2 and 3.

When the original applied stress system is considered as the sum of the two separate stress systems given, the hole can be made as near neutral as required by adopting the following procedure.

(*a*) *State* 2

Consider the plate with its elliptical hole which for the time being is unreinforced. Let it be loaded around its outer boundary with loads corresponding to a principal stress system of $(1-\omega)f$ and $(n-\omega)f$. At the same time let forces be applied to the boundary of the plate around the hole which are equal to the interaction which would occur across this boundary if no hole were present. The stresses in the plate under the total load system are therefore unaffected by the presence of the hole.

(*b*) *State* 3

Using Inglis' solution, calculate the stresses in the perforated plate for principal stresses ωf applied at infinity.

(*c*) Combining (*a*) and (*b*), obtain the stress distribution in a plate with an unreinforced elliptical hole caused by applied principal stresses f and nf at its outer boundary and by principal stresses $(1-\omega)f$ and $(n-\omega)f$ at the boundary of the hole.

(*d*) Determine the necessary cross-sectional area of the elliptical reinforcement which when unstressed would exactly fit around the hole in the unstressed plate, and which when stressed by forces corresponding to the principal stresses $(1-\omega)f$ and $(n-\omega)f$ reversed has strains identical with those in the plate around the hole due to the applied stresses in item (*c*).

The external forces across the junction of the plate and reinforcement may now be regarded as having been liquidated, and the action everywhere is the same as that in a plate with a reinforced elliptical hole under the action of the applied principal stresses f and nf acting at infinity as in state 1.

When the above procedure is adopted the cross-sectional area of the reinforcement and the maximum stress in the plate are given by simple expressions containing the arbitrary parameter ω. As shown in subsequent examples, these expressions can be used to design practical reinforcements which correspond to maximum stresses only slightly greater than the maximum stress in a similarly loaded plate with no hole.

3. Theory

If φ is a function which satisfies the equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2\varphi=0 \quad . \quad . \quad . \quad (1)$$

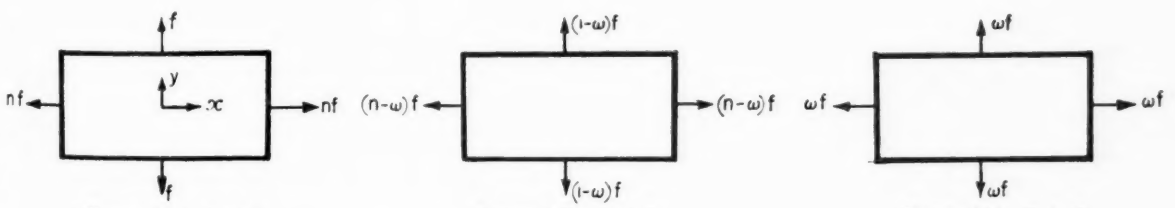

FIGURE 1(*a*). State 1. FIGURE 1(*b*). State 2. FIGURE 1(*c*). State 3.

Resolution of the applied stress system into two component applied stress systems.

the stresses in the plate are given by

$$\left.\begin{aligned}\widehat{xx}&=\frac{\partial^2\varphi}{\partial y^2}\\ \widehat{yy}&=\frac{\partial^2\varphi}{\partial x^2}\\ \widehat{xy}&=-\frac{\partial^2\varphi}{\partial x\partial y}\end{aligned}\right\} \quad . \quad . \quad (2)$$

For the stress distribution associated with the external loading in state 2

$$\varphi=(1-\omega)\frac{fx^2}{2}+(n-\omega)\frac{fy^2}{2} \quad . \quad . \quad (3)$$

Considering the equilibrium of an elemental length of reinforcement under the action of the state 2 stresses $\widehat{xx}_2=(n-\omega)f$ and $\widehat{yy}_2=(1-\omega)f$, it is given in Ref. 2 that for the hole to be neutral the force acting on a normal cross section of the reinforcement is given by

$$F=h\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]^{\frac{1}{2}}$$

i.e. $$F=hf\left[(1-\omega)^2x^2+(n-\omega)^2y^2\right]^{\frac{1}{2}} \quad . \quad (4)$$

where h is the thickness of the plate and F the force acting on a section of the reinforcement normal to the curve of the hole.

The strain in the reinforcement due to the stresses in state 2 is given in terms of the strains in the plate by the following relationship,

$$\Delta_{R2}=\Delta_{x2}\cos^2\theta+\Delta_{y2}\sin^2\theta \quad . \quad . \quad (5)$$

where θ is the angle between the x-axis and a tangent to the curve forming the hole.

There can be no appreciable shear force acting on a radial cross section of the reinforcement because its bending stiffness in the plane of the plate is small. This follows from the work of Gurney[3] who concluded that reinforcements which have a relatively small bending stress in the plane of the stressed plate experience primarily tensile and compressive stresses as opposed to bending stresses. It therefore follows that if a hole is to be neutral for a given stress system, the shape of the hole must be chosen so that the shear force on any normal cross section of the reinforcement is zero.

It has been shown by Mansfield that the necessary shape of the neutral hole for the applied stress system in state 2 is an ellipse given by the equation

$$x^2+\frac{(n-\omega)y^2}{(1-\omega)}=a^2. \quad . \quad . \quad (6)$$

where a is the semi-major axis which can be chosen arbitrarily.

From (6)

$$\tan\theta=-\frac{(1-\omega)x}{(n-\omega)y}$$

so that $$\sin^2\theta=\frac{(1-\omega)^2x^2}{\left[(n-\omega)^2y^2+(1-\omega)^2x^2\right]}$$

and $$\cos^2\theta=\frac{(n-\omega)^2y^2}{\left[(n-\omega)^2y^2+(1-\omega)^2x^2\right]} \quad . \quad (7)$$

Now $$\Delta_{x2}=\frac{f}{E}\left[(n-\omega)-\sigma(1-\omega)\right]$$

and $$\Delta_{y2}=\frac{f}{E}\left[(1-\omega)-\sigma(1-\omega)\right] \quad . \quad (8)$$

where E is Young's modulus and σ Poisson's ratio.

From equations (5), (7) and (8)

$$\frac{E\Delta_{R2}}{f}\left[(n-\omega)^2y^2+(1-\omega)^2x^2\right]$$
$$=\left[(n-\omega)-\sigma(1-\omega)\right](n-\omega)^2y^2+$$
$$+\left[(1-\omega)-\sigma(n-\omega)\right](1-\omega)^2x^2 \quad . \quad . \quad (9)$$

Let the cartesian co-ordinates x and y be functions of the variables ξ and η so that

$$\left.\begin{aligned}y&=c\cosh\xi\cos\eta\\ x&=c\sinh\xi\sin\eta\end{aligned}\right\} \quad . \quad . \quad (10)$$

where c is a constant.

From these equations two further equations can be obtained, namely

$$\frac{x^2}{c^2\cosh^2\xi}+\frac{y^2}{c^2\sinh^2\xi}=1 \quad . \quad . \quad (11)$$

and $$\frac{x^2}{c^2\cos^2\eta}-\frac{y^2}{c^2\sin^2\eta}=1 \quad . \quad . \quad (12)$$

For different constant values of ξ, equation (11) gives a family of confocal ellipses having the semi-axes $c\cosh\xi$ and $c\sinh\xi$. Also, for different constant values of η, equation (12) gives a family of confocal hyperbolas. The intersection of an ellipse and hyperbola is always at right angles, and the co-ordinate system formed is said to be elliptic. The solution of the problem specified in state 3 can be obtained by using these co-ordinates (as shown in Ref. 4, page 197). In this case the stresses act in the directions shown in Fig. 2.

Since the reinforcement is designed so that it has no effect on the stress distribution due to the application of the principal stress system of state 3, the boundary conditions are

$\widehat{xx}_3=\widehat{yy}_3=\omega f$ at infinity.

$\widehat{\xi\xi}_3=\widehat{\xi\eta}_3=0$ on the boundary of the hole.

Using the solution obtained by Inglis and given in Ref. 4, it can be shown that around the hole

$$(\widehat{\eta\eta}_3)_{\xi=\xi_0}=\frac{2\omega f\sinh 2\xi_0}{\cosh 2\xi_0-\cos 2\eta} \quad . \quad . \quad (13)$$

From equations (10)

$$\left.\begin{aligned}\sinh 2\xi_0&=\frac{2ab}{c^2}\\ \cosh 2\xi_0&=\frac{a^2+b^2}{c^2}\end{aligned}\right\} \quad . \quad . \quad (14)$$

where a is the semi-major axis of the ellipse measured along the x co-ordinate axis, b the semi-minor axis of

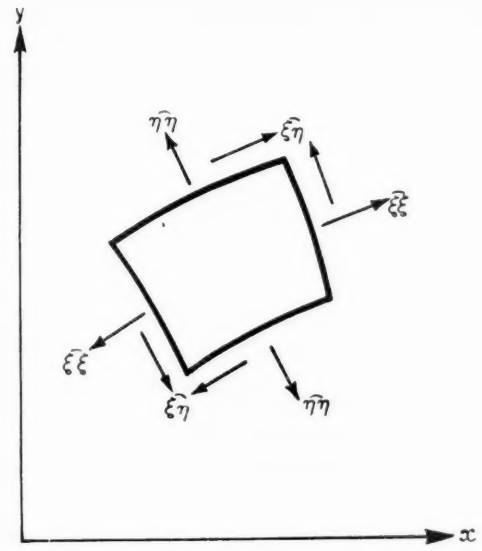

FIGURE 2. Stresses acting on an elemental area of the plate.

the ellipse measured along the y co-ordinate axis, and $c^2 = a^2 - b^2$.

Let

$$K = \frac{a}{b} = \left[\frac{(n-\omega)}{(1-\omega)}\right]^{\frac{1}{2}} \quad . \quad . \quad (15)$$

Then (14) becomes

$$\left.\begin{aligned} \sinh 2\xi_0 &= \frac{2K}{K^2-1} \\ \cosh 2\xi_0 &= \frac{K^2+1}{K^2-1} \end{aligned}\right\} \quad . \quad . \quad (16)$$

since $\cos\eta = x/c\cosh\xi_0$ around the hole, it can be shown that

$$\cos 2\eta = \frac{2x^2 - b^2K^2}{b^2K^2} \quad . \quad . \quad . \quad (17)$$

From (13), (16) and (17) the stress distribution around the hole is given by

$$(\widehat{\eta\eta})_{\xi=\xi_0} = \frac{2\omega a^2K(1-\omega)f}{a^2(n-\omega) - x^2(n-1)} \quad . \quad . \quad (18)$$

Thus the strain in the reinforcement corresponding to the stresses in state 3 is given by the following expression:

$$\frac{E\Delta_{R3}}{f} = \frac{2\omega a^2K(1-\omega)}{a^2(n-\omega) - x^2(n-1)} \quad . \quad . \quad (19)$$

As the reinforcement is chosen so that it has a negligible bending stiffness in the plane of the plate, its width will necessarily be small compared with the lateral dimensions of the hole. This implies that the strain in the reinforcement is constant over a normal cross section, so that for state 1

$$\Delta_{R1} = \frac{F}{A_R E} \quad . \quad . \quad . \quad (20)$$

where A_R is the cross-sectional area of the reinforcement at any particular point.

From equations (4) and (20)

$$\frac{E\Delta_{R1}}{f} = \frac{h}{A_R}\sqrt{[(1-\omega)^2x^2 + (n-\omega)^2y^2]} \quad . \quad (21)$$

Now

$$\Delta_{R1} = \Delta_{R2} + \Delta_{R3} \quad . \quad . \quad . \quad (22)$$

Substituting (5), (9) and (20) in (22), the following expression is obtained for the necessary cross-sectional area of the reinforcement:

$$A_R\left(\frac{[(n-\omega)-\sigma(1-\omega)](n-\omega)^2y^2 + [(1-\omega)-\sigma(n-\omega)](1-\omega)^2x^2}{[(n-\omega)^2y^2 + (1-\omega)^2x^2]} + \right.$$
$$\left. + \frac{2\omega a^2K(1-\omega)}{[(n-\omega)a^2 - (n-1)x^2]}\right) = h\sqrt{[(1-\omega)^2x^2 + (n-\omega)^2y^2]} \quad . \quad (23)$$

Using the equation for the ellipse to write y^2 in terms of x^2, the equation (23) can be written

$$A_R\{[(n-\omega)^2 - \sigma(1-\omega)(n-\omega) + 2K\omega(1-\omega)]a^2 - [(n-\omega)^2 - (1-\omega)^2]x^2\} = h[(n-\omega)a^2 - (n-1)x^2]^{3/2}(1-\omega)^{\frac{1}{2}} \quad . \quad (24)$$

From (24) it is seen that for a given applied stress system, the cross-sectional area of the reinforcement is a function of the parameter ω. That is, the variation in the cross-sectional area of the reinforcement can be controlled by varying the ratio of the major and minor axes of the ellipse.

If $\omega = 0$ the hole is neutral and the cross-sectional area of the reinforcement is given by

$$A_R = \frac{h[na^2 - (n-1)x^2]^{3/2}}{[(n^2 - \sigma n) - (n^2-1)x^2]} \quad . \quad . \quad (25)$$

In the important case of a boiler stress distribution $n = 2$ and equation (25) becomes

$$A_R = \frac{h[2a^2 - x^2]^{3/2}}{[2(2-\sigma)a^2 - 3x^2]} \quad . \quad . \quad (26)$$

This expression indicates that, except for small values of σ, the area of reinforcement required to make the hole neutral is too large to be of much practical use.

The tangential stress around the hole in state 2 is given by the following well known relationship,

$$(\widehat{\eta\eta}_2)_{\xi=\xi_0} = \widehat{xx}\cos^2\theta + \widehat{yy}\sin^2\theta \quad . \quad (27)$$

Substituting the appropriate expressions for $\widehat{xx}$, $\cos^2\theta$, and so on, this reduces to

$$(\widehat{\eta\eta}_2)_{\xi=\xi_0} = \frac{f[(n-\omega)^2(a^2-x^2) + (1-\omega)^2x^2]}{[(n-\omega)a^2 - (n-1)x^2]} \quad (28)$$

Adding (18) and (28) the following expression is obtained for the tangential stress around the hole in state 1.

$$(\widehat{\eta\eta}_1)_{\xi=\xi_0} = \frac{f[(n-\omega)^2(a^2-x^2) + (1-\omega)^2x^2 + 2K\omega(1-\omega)a^2]}{[(n-\omega)a^2 - (n-1)x^2]} \quad . \quad . \quad . \quad (29)$$

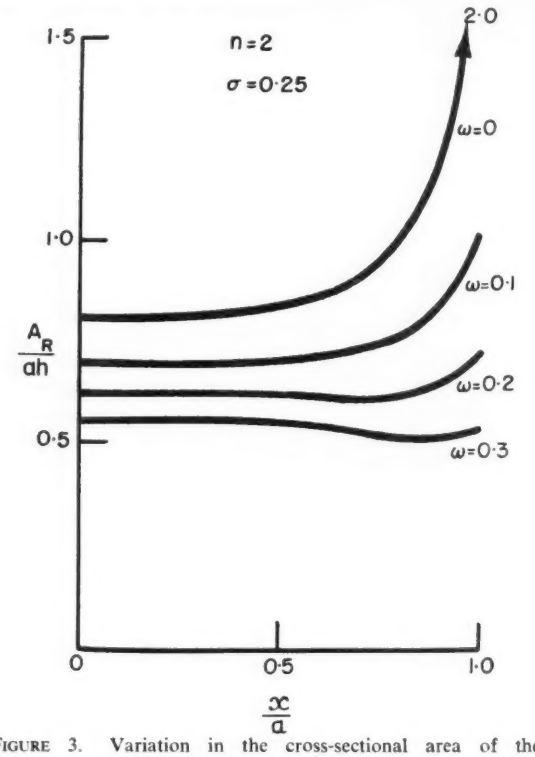

FIGURE 3. Variation in the cross-sectional area of the reinforcement when $n=2$ and $\sigma=0{\cdot}25$.

From (4), (6) and (24) the stress acting on a normal cross section of the reinforcement is given by

$$f_R\,[a^2\,(n-\omega)-x^2\,(n-1)]=$$
$$=[(n-\omega)^2-\sigma\,(1-\omega)\,(n-\omega)+2K\omega\,(1-\omega)]\,a^2-$$
$$-[(n-\omega)^2-(1-\omega)^2]\,x^2 \qquad . \quad . \quad . \quad (30)$$

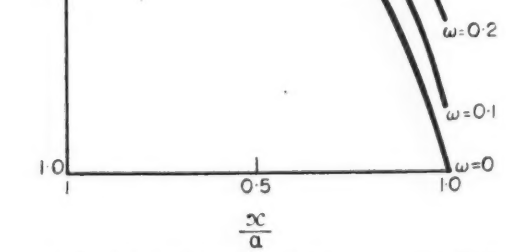

FIGURE 5. Variation in the tangential stress around the hole when $n=2$.

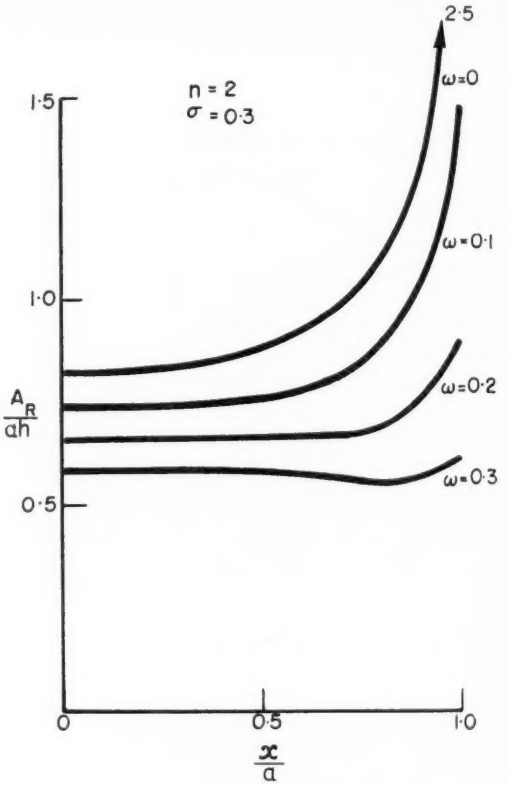

FIGURE 4. Variation in the cross-sectional area of the reinforcement when $n=2$ and $\sigma=0{\cdot}3$.

As a numerical example, the particular case of $n=2$ is taken and the cross-sectional area of the reinforcement, the tangential stress in the plate around the hole, and the stress on a normal cross section of the reinforcement evaluated for different values of ω and practical values of σ. It is shown in Figs. 3, 4, 5, 6 and 7 that a reinforcement corresponding to a stress increase of 4·5 per cent can be designed by giving ω a value in the order of 0·3. For such a value of ω the variation in the cross-sectional area of the reinforcement is small and for practical purposes can be neglected.

It is worth noting that since the hole is neutral for the stress system shown in state 2, any deviation in the original stress system due to the introduction of the hole is associated completely with the applied stress system shown in state 3. As shown by Inglis, the deviation in the state 3 stress system rapidly decreases as the distance from the hole increases and soon becomes negligible compared with the applied principal stresses. (For example, in the case of a circular hole of radius a the tangential stress at a distance r from its centre is given by $f\,(1+a^2/r^2)$, which becomes f when r is several times greater than a.) This implies that the stress deviation in state 1 is confined to a small area around the hole since, for practical values of ω, the state 2 stresses predominate and therefore further reduce

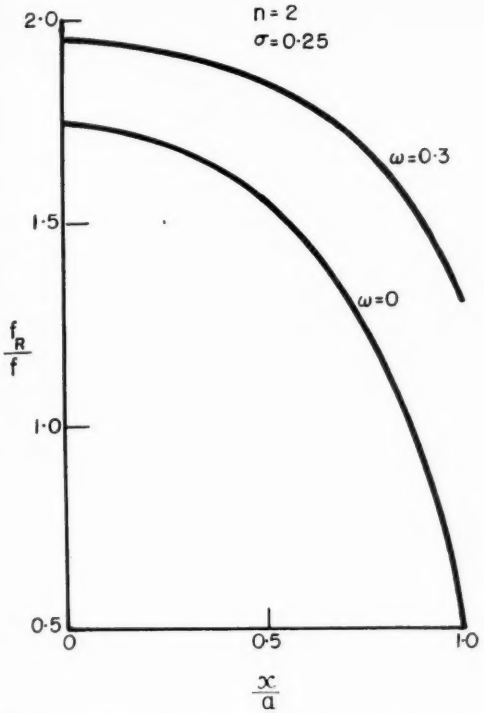

FIGURE 6. Stress on a normal cross section of the reinforcement when $n=2$ and $\sigma=0{\cdot}25$.

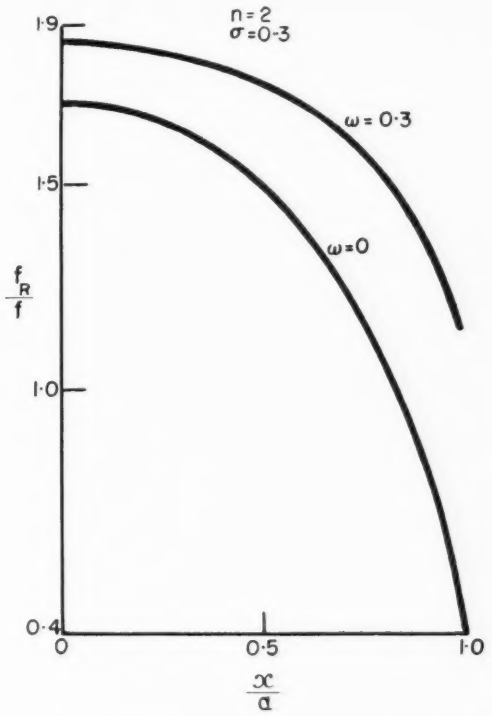

FIGURE 7. Stress on a normal cross section of the reinforcement when $n=2$ and $\sigma=0{\cdot}3$.

the area around the hole where the total stress deviation is greater than a given value.

Thus one can assume that the solution for an infinite plate is correct for a plate of finite size and applicable to a system of variable principal stresses (for example, ellipsoidal shells) provided that the variation in the principal stresses, in the area where it is proposed to introduce the hole, is sufficiently small to be considered negligible.

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TECHNICAL NOTES

Response of Missile Erector Towers to Sudden Stops or Impact Loads

by

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THE CANTILEVER-TYPE mechanism often used in large missile erectors is subjected to heavy stresses when raising a missile from the horizontal to the vertical position for launching. In the event it becomes necessary to return the missile to the horizontal position, severe stresses in the erector, and possibly the missile itself may be developed if the lowering mechanism causes the erector to stop suddenly just before it reaches the horizontal position. This note presents a simple matric formulation of the problem which enables one who has access to an electronic digital computer to compute maximum deflections and load factors that occur as a result of a sudden stop.

NOTATION

a_{ij}	deflection at i resulting from unit load at j
F_{total}	total maximum force at each mass when the maximum deflections occur
m_i	mass of tower or missile at i^{th} station
$Q(t)$	sudden impulse applied at line AA
r_i	distance from hinge to i^{th} mass
T_{total}	total kinetic energy initially put into the system
t	time
t_1	time at which impulse $Q(t)$ stops
v_r	rotational velocity of structure about hinge
W_n	work done for displacements corresponding to an n^{th} mode
x_{in}	transverse displacement of i^{th} mass when system is excited into the n^{th} natural frequency
z_{in}	maximum amplitude of i^{th} mass vibrating at n^{th} natural frequency
τ	fundamental natural period of the structure
ω_n	n^{th} natural circular frequency

SUDDEN STOP

For engineering purposes, a sudden stop may be considered as any stop of tower rotational velocity that takes less than $1/(2\pi)$ times the natural fundamental period of the structure. For example, to obtain an estimate of the error involved in making the above assumption, consider a massless cantilever beam with an attached mass at the end of the beam; the beam is rotating about the fixed end so that the tip mass travels along a circular arc. If the rotational velocity of the beam is diminished by linear angular deceleration so that the rotational velocity becomes zero in a time equal to $1/(2\pi)$ times the natural period of the system, the maximum displacement may be computed exactly for the elastic system by the Duhamel Integral. If this exact displacement value is compared with the value obtained by assuming instantaneous stopping of rotational velocity, a difference of 4·16 per cent occurs; the instantaneous stopping gives, of course, the larger displacement.

Consider the elastic structure shown in Fig. 1. The tower is assumed to bend about the principal yy axis. The tower rotational velocity is stopped suddenly so that

$$\text{time to stop} \leqq \tau/2\pi \quad . \quad . \quad (1)$$

Received 17th June 1957.

where τ is the fundamental natural period of the structure. The base of the tower spring support (line AA Fig. 1) is assumed to rotate with the tower, and the method of applying a sudden stop to the tower is to stop the movement of the base of the tower spring support. The effects of centrifugal force (resulting from rotation of the tower) on producing longitudinal motions of the tower and missile are assumed to be negligible; the primary motions of the structure are in the flexible transverse direction. For a very rapid stop, the time to reach maximum displacements will be approximately $\tau/4$.

The masses of the tower and missile are lumped at a finite number of points as shown in Fig. 1. The masses of the tower and missile are numbered consecutively from m_1 to m_q. When the tower is stopped suddenly all of the natural modes of the missile-tower system will be excited; however, almost all of the available kinetic energy will be used up in exciting the lower modes. For many problems, in which the separation of the lowest two natural frequencies is similar to a uniform cantilever beam, over 98 per cent of the available kinetic energy will be utilised in producing a deflection shape composed largely of the first natural mode shape and a small component of the second natural mode shape. The relative deflections that the system will take when stopped suddenly may be derived as follows: Let

$$x_{in} = z_{in} \sin \omega_n t \quad . \quad . \quad . \quad (2)$$

where

x_{in}	transverse displacement of i^{th} mass when system is excited into the n^{th} natural frequency
ω_n	n^{th} natural circular frequency
z_{in}	maximum amplitude of i^{th} mass vibrating at n^{th} natural frequency
t	time.

If the system were vibrating freely in the n^{th} natural mode shape, it would contain the following maximum total kinetic energy:

$$T_n = \frac{\omega_n^2}{2} \sum_{i=1}^{q} m_i z_{in}^2 \quad . \quad . \quad (3)$$

For a sudden stop it is desired to find the energy required to excite each mode so that the value of the summation in equation (3) will remain constant. Obviously the amounts of energy required to produce equal values of the summation in equation (3) vary with the squares of the natural frequencies, and, if the lower frequencies are widely separated, most of the available kinetic energy resulting from sudden stopping will go into exciting the fundamental mode. For the problem under consideration assume that only the lowest two natural modes are excited. Therefore,

$$T_1 = \frac{T_{\text{total}}}{1 + \left(\frac{\omega_1}{\omega_2}\right)^2} \quad . \quad . \quad . \quad . \quad (4)$$

$$T_2 = \frac{T_{\text{total}}}{1 + \left(\frac{\omega_2}{\omega_1}\right)^2} = T_{\text{total}} - T_1 \quad . \quad (5)$$

and, $$T_{\text{total}} = \frac{1}{2} \sum_{i=1}^{q} m_i (\dot{x}_i)^2 \quad . \quad . \quad . \quad (6)$$

where T_{total} is the total kinetic energy that is put initially into the system; the $\dot{x}_i$ values may be computed from the angular velocity of the system just before the stopping begins.

The two lowest natural frequencies and mode shapes may be computed easily from the familiar matric form

where, $$(AM - \lambda_n I)\{\alpha_n\} = 0 \quad . \quad . \quad (7)$$

$$A = \begin{bmatrix} a_{11}a_{12}\ldots a_{1q} \\ a_{21}a_{22}\ldots a_{2q} \\ \cdot \quad \cdot \\ \cdot \quad \cdot \\ a_{q1}a_{q2}\ldots a_{qq} \end{bmatrix}$$

= matrix of influence coefficients where a_{ij} = deflection at i resulting from a unit load at j

$$M = \begin{bmatrix} m_1 & & & & & & \\ & m_2 & & & & & \\ & & \cdot & & & & \\ & & & \cdot & & & \\ & & & & m_i & & \\ & & & & & \cdot & \\ & & & & & & \cdot \\ & & & & & & & m_q \end{bmatrix}$$

= diagonal mass matrix

$$\{\alpha_n\} = \begin{Bmatrix} \alpha_{1n} \\ \alpha_{2n} \\ \cdot \\ \cdot \\ \alpha_{in} \\ \cdot \\ \cdot \\ \alpha_{qn} \end{Bmatrix}, \quad \lambda_n = \frac{1}{\omega_n^2}$$

The sum of the kinetic energies T_1 and T_2 must equal the total work done as the masses move to their maximum deflected positions. Thus, it will be necessary to get the forces at the masses as functions of displacements and then integrate the forces times the displacements. The displacements at each mass may be written in terms of the influence coefficients and forces as

$$\begin{aligned} x_{1n} &= a_{11}F_{1n} + a_{12}F_{2n} + \ldots + a_{1q}F_{qn} \\ &\cdot \\ &\cdot \\ x_{in} &= a_{i1}F_{1n} + a_{i2}F_{2n} + \ldots + a_{iq}F_{qn} \quad . \quad (8) \\ &\cdot \\ &\cdot \\ x_{qn} &= a_{q1}F_{1n} + a_{q2}F_{2n} + \ldots + a_{qq}F_{qn} \end{aligned}$$

Therefore, $$F_n = A^{-1}X_n \quad . \quad . \quad . \quad . \quad (9)$$

$$F_n = \begin{Bmatrix} F_{1n} \\ F_{2n} \\ \cdot \\ \cdot \\ F_{in} \\ \cdot \\ \cdot \\ F_{qn} \end{Bmatrix}, \quad X_n = \begin{Bmatrix} x_{1n} \\ x_{2n} \\ \cdot \\ \cdot \\ x_{in} \\ \cdot \\ \cdot \\ x_{qn} \end{Bmatrix}$$

and the subscript n refers to forces and displacements associated with the n^{th} natural mode. Let

$$x_{in} = \alpha_{in}x_{1n}, \quad i = 1, 2 \ldots q \quad . \quad . \quad (10)$$

$$n = 1, 2$$

where $\alpha_{1n} = 1{\cdot}0$, dimensionless here.

Substituting the values of x_{in} from equation (10) into equation (9) gives

$$F_n = x_{1n}A^{-1}\{\alpha_n\} = x_{1n}\{c_n\} \quad . \quad . \quad (11)$$

where $$\{c_n\} = A^{-1}\{\alpha_n\} = \begin{Bmatrix} c_{1n} \\ c_{2n} \\ \cdot \\ \cdot \\ c_{in} \\ \cdot \\ \cdot \\ c_{qn} \end{Bmatrix}$$

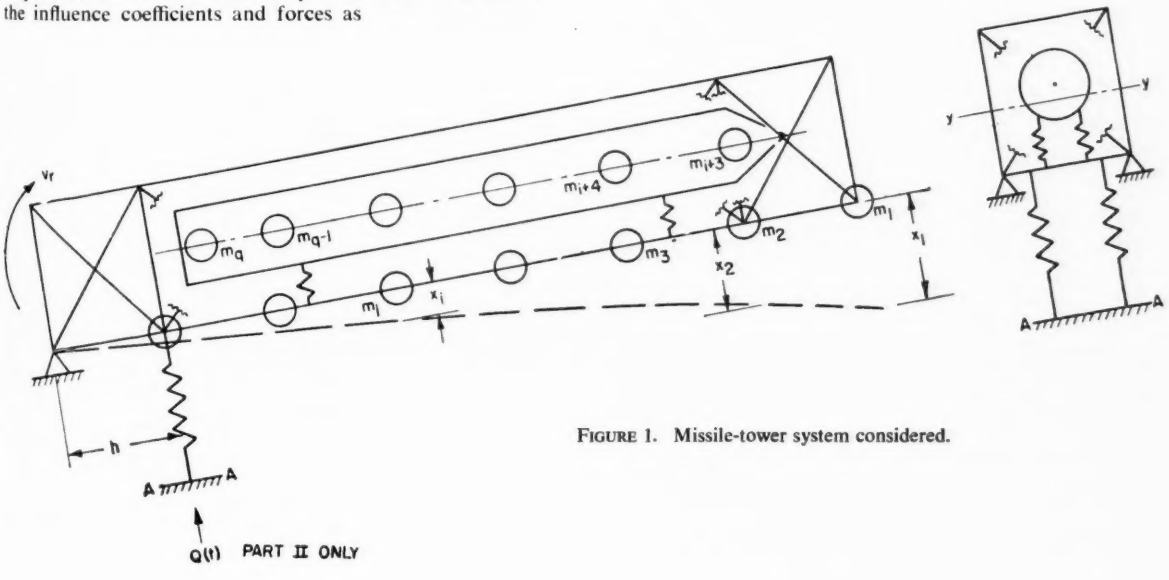

FIGURE 1. Missile-tower system considered.

From equation (10)

$$dx_{in}=\alpha_{in}dx_{1n}$$

and from equations (9) and (11)

$$F_{in}=c_{in}x_{1n}$$

Therefore the work done for displacements corresponding to an n^{th} mode will be

$$W_n=\sum_{i=1}^{q}\int_0^{x_{in}}F_{in}dx_{in}=\sum_{i=1}^{q}\int_0^{x_{1n}}\alpha_{in}c_{in}x_{1n}dx_{1n}$$

$$=\tfrac{1}{2}\sum_{i=1}^{q}\alpha_{in}c_{in}x_{1n}^2 \quad . \quad . \quad . \quad (12)$$

Equating the values of W_1 and W_2 from equation (12) to the values of T_1 and T_2 from equations (4) and (5) gives

$$x_{11}=\sqrt{\left\{\frac{\sum_{i=1}^{q}m_i(\dot{x}_i)^2}{\left[1+\left(\frac{\omega_1}{\omega_2}\right)^2\right]\sum_{i=1}^{q}\alpha_{i1}c_{i1}}\right\}} \quad . \quad (13)$$

$$x_{12}=\sqrt{\left\{\frac{\sum_{i=1}^{q}m_i(\dot{x}_i)^2}{\left[1+\left(\frac{\omega_2}{\omega_1}\right)^2\right]\sum_{i=1}^{q}\alpha_{i2}c_{i2}}\right\}} \quad . \quad (14)$$

The total deflection at mass one will be the sum of x_{11} and x_{12} from equations (13) and (14). Thus

$$x_{1(\text{total})}=x_{11}+x_{12}, \quad . \quad . \quad . \quad (15)$$

and the final total deflected shape will be

$$\{x_i\}=x_{11}\{\alpha_{i1}\}+x_{12}\{\alpha_{i2}\} \quad . \quad . \quad (16)$$

where x_{11} and x_{12} are computed from equations (13) and (14) respectively and $\{\alpha_{i1}\}$ and $\{\alpha_{i2}\}$ are the first and second natural mode shapes as determined from equation (7). (Note that $\alpha_{11}=\alpha_{12}=1{\cdot}0$. Dimensionless in equation (16).)

From equation (11) the total maximum force at each mass, when the maximum deflections occur, may be written as

$$F_{\text{total}}=F_1+F_2=x_{11}\{c_1\}+x_{12}\{c_2\} \quad . \quad (17)$$

Equation (17) gives the dynamic forces on the system which, of course, must be added to the dead load forces when shears, moments and stresses are to be computed.

RESPONSE TO IMPULSE AT BASE

If the structure shown in Fig. 1 is at rest and a sudden impulse force $Q(t)$ is applied at line AA and the duration of the force $Q(t)$ is less than $\tau/(2\pi)$, one may compute the approximate response of the structure by assuming that the structure attains an instantaneous rotational velocity v_r. Equating angular impulse to the change in angular momentum gives

$$\int_0^{t_1}Q(t)\,hdt=v_r\sum_{i=1}^{q}m_ir_i^2$$

Therefore

$$\dot{x}_i=r_iv_r=r_i\frac{\int_0^{t_1}Q(t)\,hdt}{\sum_{i=1}^{q}m_ir_i^2} \quad . \quad . \quad (18)$$

where r_i distance from hinge to i^{th} mass
v_r rotational velocity of structure about hinge
t_1 time at which impulse $Q(t)$ stops

and, of course, $Q(t)$ may be arbitrary so long as $t_1\leqq\tau/(2\pi)$. (For approximate analysis.)

The values of $\dot{x}_i$ from equation (18) may be used in equations (13) and (14) for computing the maximum end deflection, equation (15), for the structure. Then, the dynamic forces may be computed from equation (17).

CONCLUDING REMARKS

Although several simplifying assumptions have been made, an analysis by the method outlined here should give results that are sufficiently accurate for engineering purposes. For preliminary analysis, the truss may be considered as a beam in which the chord members carry the flexure and the diagonals the shear. The shear influence coefficients may be computed easily and the flexural influence coefficients may be computed rapidly by hand by Newmark's method. The method of Newmark[1] is particularly adapted to computing influence coefficients of variable stiffness beams since the deflections at all of the selected mass points may be obtained easily for a load applied at one particular mass point.

For final or more accurate analysis the influence coefficients may be computed conveniently on an electronic digital computer by the method of Denke[2]; the system may be determinate or indeterminate.

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Shear Moduli for Flat Panels and the Effect of Flange Flexibility

by

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THIS NOTE describes an investigation into the secant and tangent moduli for flat panels in shear. Consideration is given to a comparatively recent analysis giving the secant modulus and comparison is made with he Society's Data Sheets. The effect of flange flexibility is investigated, and the analysis is extended to cover e tangent modulus.

Received May 1957.

NOTATION

t	plate thickness
a	depth of panel
b	stiffener spacing
A_s, A_f	cross-sectional area of stiffener and flange
q	nominal shear stress in plate
q_b	shear stress at which plate first buckles
ε	direct strain in the plate in the direction of diagonal tension

ε_s, ε_f direct strain in stiffener and in flange due to diagonal tension field in plate
E, G elastic modulus and shear moduli of material of plate
σ Poisson's ratio of material of plate
G_s, G_t secant and tangent shear modulus of panel
k diagonal tension factor
α angle of diagonal tension
B flange flexibility parameter

INTRODUCTION

The Royal Aeronautical Society is at present undertaking a review of the 02.03 series of Structures Data Sheets[1] dealing with flat panels in shear. This series of sheets was prepared some years ago and since that time an appreciable amount of new data has become available. Probably the most significant of recent published reports on the topic is that by Kuhn, Peterson and Levin[2] in which a considerable quantity of experimental data is used in deriving a semi-empirical method of analysis. (Reference 2 has already been used in the Data Sheets as the main basis for the data on curved panels in shear.)

It is appropriate therefore to consider this method of analysis and to compare it with the existing Data Sheets. In this note the comparison deals in particular with the secant and tangent shear moduli of flat panels, but it is probable that the general conclusions will also apply to other relevant parameters such as flange loading, stiffener loading, and so on.

SOME ASPECTS OF THE ANALYSIS OF N.A.C.A. T.N.2661

A fundamental feature of the analysis of N.A.C.A. T.N.2661 is the division of the applied nominal web shear stress, q, into two parts, a shear stress q_{SH} carried by true shear action of the web, and a part q_{DT} carried by diagonal tension action. Thus

$$q = q_{SH} + q_{DT} \text{ where } q_{DT} = kq$$

and the diagonal tension factor k is given empirically by equation (1).

$$k = \tanh\left[0{\cdot}5 \log_{10}(q/q_b)\right] \quad . \quad . \quad (1)$$

In the same way the total shear deformation of the panel, γ, is split into two parts

$$\gamma = \gamma_{SH} + \gamma_{DT}$$

For the determination of the deflection of a shear panel under load, the effective shear modulus is required. This modulus is given by q/γ and, in the manner of the Data Sheets, is termed the "secant shear modulus of the panel" and denoted by G_s. From the relations given above it can be shown that,

$$\frac{1}{G_s} = \frac{1-k}{G} + \frac{k}{G_{DT}}$$

If the plate, stiffeners and flanges are considered to be of the same material, strain energy considerations then lead to the relation for G_s, given by equation (2),

$$\frac{G}{G_s} = 1 - k + \frac{k}{2(1+\sigma)}\left[\frac{4}{\sin^2 2\alpha} + \frac{\tan^2\alpha}{\frac{A_s}{bt} + 0{\cdot}5(1-k)} + \frac{\cot^2\alpha}{\frac{2A_f}{at} + 0{\cdot}5(1-k)}\right] \quad . \quad . \quad (2)$$

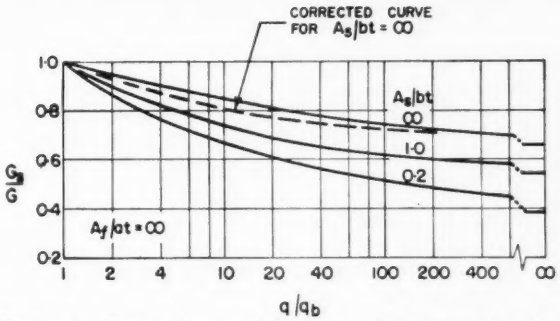

FIGURE 1. Secant shear modulus curves from T.N.2661 with correction.

The angle of diagonal tension, α, is given by equation (3),

$$\tan^2\alpha = \frac{\varepsilon - \varepsilon_f}{\varepsilon - \varepsilon_s} \quad . \quad . \quad . \quad . \quad (3)$$

and may be determined by a method of successive approximation from the appropriate expressions for ε, ε_f and ε_s given in T.N.2661.

For curved shear panels the angle of diagonal tension is given by a relation similar to that for flat panels, and a graphical method for determining its value is given in Data Sheet 02.03.22. This data sheet can be applied to flat panels by taking the radius of the panel as infinite.

In T.N.2661 Poisson's ratio is taken as 0·33 and a set of curves is presented giving G_s/G against q/q_b for various values of $A_s/(bt)$ and for the particular case of heavy flanges, *i.e.* $A_f/(at)$ is taken as infinite. It must be mentioned here that the curve for $A_s/(bt) = \infty$, ($A_{ue}/(dt) = \infty$ in the notation of T.N.2661) appears to be in error by amounts up to about 5 per cent, whereas the remaining curves ($A_s/(bt) = 1{\cdot}0$, 0·4 and 0·2) agree closely with values computed from the appropriate equations. The curve for $A_s/(bt) = \infty$ is, of course, correct at $q/q_b = 1{\cdot}0$, and it is also correct at $q/q_b = \infty$. A reproduction of the relevant figure from T.N.2661 is given in Fig. 1 together with the recalculated curve for $A_s/(bt) = \infty$.*

It is interesting to note that in an earlier report by Kuhn and Peterson[3] this error does not occur and the $A_s/(bt) = \infty$ curve agrees with computations from the equations. The erroneous curve has, however, been carried forward to Fig. 3.10*a* of "Stresses in Aircraft and Shell Structures" (1956) by Kuhn.

Figure 2 shows the values of G_s/G determined from T.N.2661 but with Poisson's ratio taken as 0·30 as is usual in the Data Sheets. The curves are for $A_f/(at) = \infty$. For comparison the corresponding curves from Data Sheet 02.03.06 are shown dotted, and it is seen that the new analysis indicates rather greater panel stiffness. It has been found that the new curves give closer agreement with available experimental data.

EFFECT OF FLANGE AREA

The curves of Fig. 2 were determined from equation (2) taking $A_f/(at) = \infty$ and in many practical cases the flanges are such that this simplification is justified. However, it is worthwhile investigating the effect of finite values of this parameter, since in some designs the flange area may be quite small.

The computation is straightforward provided the assumption is made that although the flanges are finite

*Since this note was written, Paul Kuhn has agreed that the curve was in error.—J.C.S.

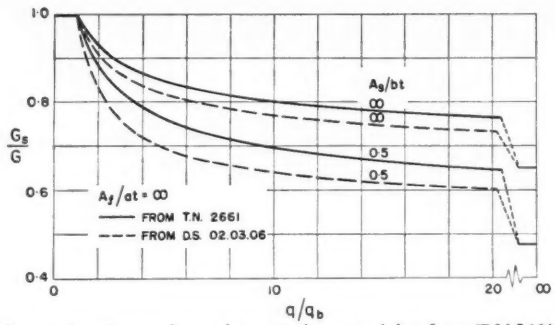

FIGURE 2. Comparison of secant shear modulus from T.N.2661 and Data Sheet (heavy flanges).

in area they remain rigid in bending. This assumption is made for two reasons. Firstly, equations (2) and (3) both depend on strain energy relations, and bending flexibility of the flanges would introduce additional strain energy terms thus modifying them. This first point is considered in more detail later. Secondly, flexibility of the flanges in the plane of the panel would mean that with the beam under shear load the flanges would deflect inwards at the middle of each bay as in Fig. 3(*a*). This in turn would influence the diagonal tension field in the beam, as shown diagrammatically in Fig. 3(*b*), with the tension diagonals being concentrated towards the stiffeners. An essential feature of the analysis of T.N.2661 is that the tension field is assumed uniform across the panel and the distribution of Fig. 3(*b*) is not considered. A non-uniform field makes the analysis extremely complex and throughout this note it is assumed that the flange deflections have negligible effect on the tension field distribution.

With this assumption introduced, Fig. 4 shows the effect of varying the flange area for the particular case of $q/q_b = 10$. Similar curves are obtained for other ratios of q/q_b. It can be seen that reduction of $A_f/(at)$ has only small effect at first, but when its value is reduced below 1·0 or 0·5, say, the effect is more marked. $A_f/(at) = 0$ is included as a limiting condition, although this case is, of course, incompatible with the assumption made.

From curves such as Fig. 4 it can be seen that if a discrepancy of say one per cent is acceptable, then the results for $A_f/(at) = \infty$ may be applied where $A_f/(at)$ is 10 or greater.

EFFECT OF FLANGE FLEXIBILITY

The derivation of the expressions for secant shear modulus, equation (2), and angle of diagonal tension, equation (3), is dependent upon strain energy relations. These relations only include the terms due to direct or shear loading in the various components and no account is taken of the bending strain energy of the flanges. This latter is now investigated.

As previously mentioned, it is assumed that the inward deflection of the flanges is insufficient to cause the tension field to be modified to the form shown in Fig. 3(*b*). Deflection of the flanges would, however, transfer towards

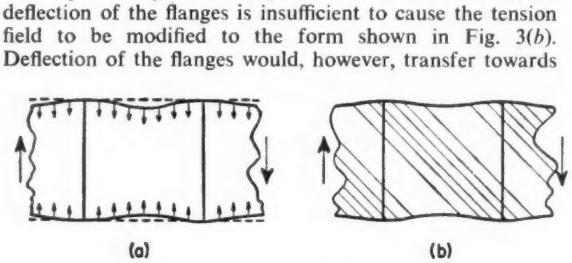

FIGURE 3. Effect of flange flexibility on tension field.

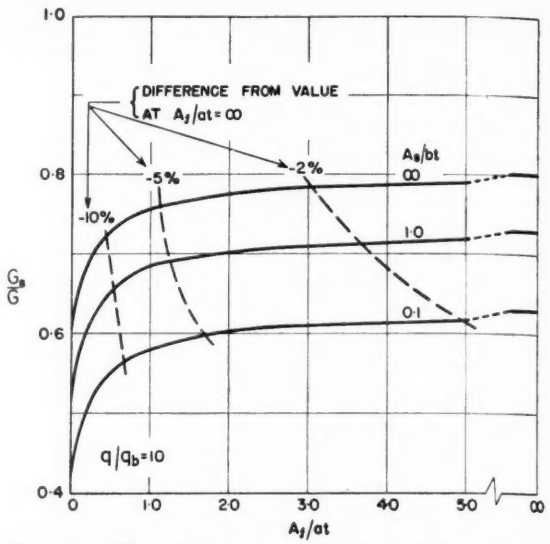

FIGURE 4. Effect of $A_f/(at)$ on G_s/G determined at $q/q_b = 10$.

the stiffeners the inward bending load on the flanges. Thus the bending moment on the flanges would be relieved by this deflection. For this reason, and within the limits of the assumptions made, the following analysis introducing the bending strain energy terms will tend to over-estimate the effects of flange bending.

From the analysis of T.N.2661 it can be shown that the component of load on the flange causing bending is,

$$f_s t\left[\frac{A_s}{bt} + 0{\cdot}5\,(1-k)\right] \text{ per unit length of flange}$$

where f_s = stress in stiffener.

Assuming the flanges to be continuous over the stiffeners, a simple integration shows that the bending strain energy per flange is equal to,

$$\frac{f_s^2 b^3\,[A_s + 0{\cdot}5\,(1-k)\,bt]^2}{1440\,EI_f}$$

where I_f = second moment of flange cross-sectional area about an axis through its centroid normal to the panel.

It should be noted that this definition for I_f may not be sufficiently complete in some cases. When end load in the flange is being considered, part of the web may be assumed to act with the flange in resisting this load, and in T.N.2661 the effective area of each flange is taken to be,

$$A_f + 0{\cdot}25\,(1-k)\,at.$$

It would be reasonable to expect, therefore, that some part of the web would also act with the flange in resisting bending (in the plane of the panel), although in most cases the magnitude of the effect on I_f would be negligible. There are cases, however, where the effect could be quite marked as, for instance, with a wing rib where a relatively thin skin might act as the flange. As yet there would seem to be insufficient experimental data to suggest how much of the web should be included in determining I_f.

Since the strain energy due to flange bending may be added directly to that due to end load, the flange bending term may be introduced directly into the appropriate energy equations. Considering the *Pure Diagonal Tension* ($k = 1$) case first, the resulting expression for α is,

$$\tan^2\alpha = \frac{\varepsilon - \varepsilon_f}{\varepsilon - \varepsilon_s\left\{1 + \dfrac{A_s b^3}{360\,I_f a}\right\}}$$

and G_s is given by,

$$\frac{G}{G_s}=\frac{1}{2(1+\sigma)}\left[\frac{4}{\sin^2 2\alpha}+\frac{bt}{A_s}\tan^2\alpha\left\{1+\frac{A_s b^3}{360\,I_f a}\right\}+\right.$$
$$\left.+\frac{at}{2A_f}\cot^2\alpha\right]$$

The corresponding expressions for the *Incomplete Diagonal Tension* case ($k<1$) are obtained from the pure diagonal tension results, as in T.N.2661, by the introduction of the factor k and by the addition of an appropriate proportion of the web to both the stiffener and flange areas. This leads to equations (4) and (5),

$$\tan^2\alpha=\frac{\varepsilon-\varepsilon_f}{\varepsilon-\varepsilon_s\{1+B\}} \quad . \quad . \quad . \quad (4)$$

$$\frac{G}{G_s}=1-k+\frac{k}{2(1+\sigma)}\left[\frac{4}{\sin^2 2\alpha}+\frac{\{1+B\}\tan^2\alpha}{\frac{A_s}{bt}+0{\cdot}5(1-k)}+\right.$$
$$\left.+\frac{\cot^2\alpha}{\frac{2A_f}{at}+0{\cdot}5(1-k)}\right] \quad . \quad (5)$$

where $$B=\frac{[A_s+0{\cdot}5(1-k)\,bt]\,b^3}{360\,I_f a} \quad . \quad . \quad . \quad (6)$$

The expressions for the components of strain occurring in equation (4) are as follows,

$$\varepsilon=\frac{q}{E}\left[\frac{2k}{\sin 2\alpha}+(1-k)(1+\sigma)\sin 2\alpha\right]$$

$$\varepsilon_f=-\frac{1}{E}\left[\frac{kq\cot\alpha}{\frac{2A_f}{at}+0{\cdot}5(1-k)}\right]$$

$$\varepsilon_s=-\frac{1}{E}\left[\frac{kq\tan\alpha}{\frac{A_s}{bt}+0{\cdot}5(1-k)}\right]$$

Comparison of equations (4) and (5) with equations (2) and (3) shows that the effect of flange bending is covered by the additional term $\{1+B\}$ in these equations, and that taking $B=0$ corresponds to neglecting the flange bending effect.

In order to investigate the magnitude of this flange bending effect, G_s has to be calculated for actual values of the parameter B. It has been found that B can have a very wide range and, for instance, where the shear beam is a wing rib with a relatively thin skin acting as the flange, very high values may be found. As previously mentioned, however, in such cases there is doubt over the appropriate values of I_f to be taken, and this could have considerable influence on the value of B.

It has been decided, therefore, to restrict this discussion to the more conventional shear beam configurations and thus values of B have been determined for shear beams of this type which have been tested experimentally. Such tests are reported by Kuhn, Peterson and Levin in N.A.C.A. T.N.2662[4], and also by Crowther and Sanderson[5]. Values of B as low as 0·0002 occurred, but it is naturally the larger values which are of more interest and at the higher end values of about 0·5 have been found.

TABLE I.

q/q_b	k	"Average" B	"Maximum" B
1·0	0	0·15	0·70
4·0	0·292	0·12	0·65
10·0	0·462	0·10	0·60
∞	1·0	0·05	0·50

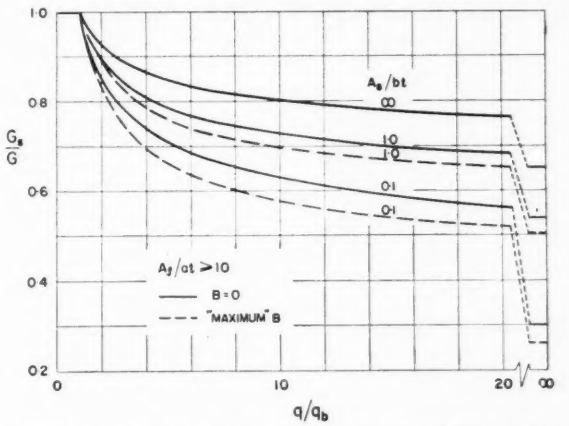

FIGURE 5. Effect of flange flexibility on secant shear modulus (heavy flanges).

In view of the wide range of values, it was decided to investigate the effect of an "average" value and also a "maximum" value, this "maximum" value being somewhat larger than that actually encountered with the test beams. The cases investigated are defined in Table I, and it should be noted that the value of B varies with q/q_b, *i.e.* with k. Since it can be shown from the equations that the limits of the effect of B will be given at the limits of the parameter $A_f/(at)$, the calculations have been carried out only for the cases $A_f/(at)\geq 10$ and $A_f/(at)=0$.

Figures 5 and 6 show the effect of B for these limiting cases and for three values of $A_s/(bt)$. It should be noted that at $A_s/(bt)=\infty$, G_s/G is unaffected by the value of B. Only the effect of "maximum" B is shown on Figs. 5 and 6 since the curve for the "average" value would be too close to that for $B=0$. However, between $B=0$ and "maximum" B the variation is roughly linear with B.

The curves show that the effect of flange bending is always to reduce the value of G_s/G. This reduction increases with decreasing $A_s/(bt)$ and it is dependent on q/q_b as well as on the value of B. For all practical cases within the ranges of the parameters covered, however, the reduction is not normally greater than 10 per cent. For

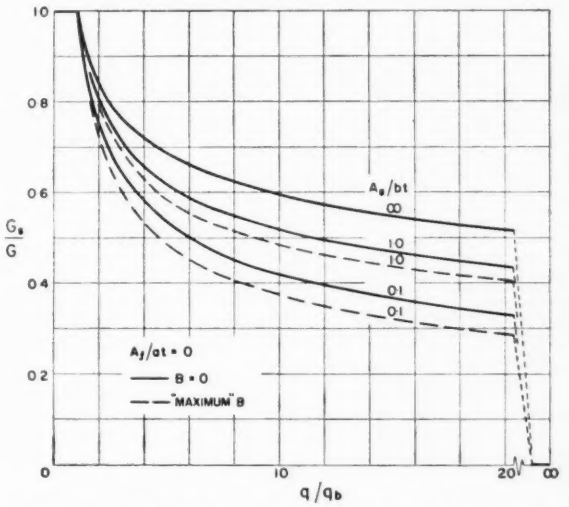

FIGURE 6. Effect of flange flexibility on secant shear modulus (very light flanges).

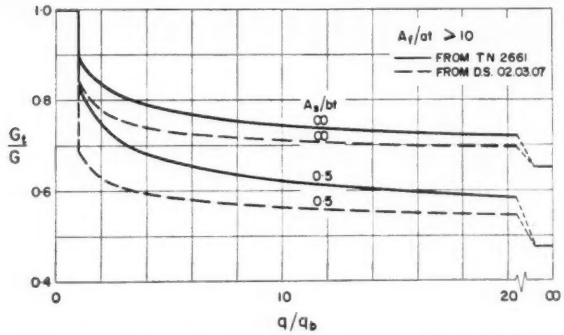

FIGURE 7. Comparison of tangent shear modulus from T.N.2661 and Data Sheet (heavy flanges).

intermediate values of the parameters, it is possible to make a reliable estimate of the effect of flange bending by interpolating between the curves given here.

It is likely that for values of B much in excess of those considered here, the flexibility of the flanges would be such as to give a tension field distribution of the form shown in Fig. 3(*b*). Thus, provided that the flange deflection is insufficient to cause appreciable non-uniformity of the tension field, Figs. 5 and 6 may be used for estimating the effect of flange bending.

TANGENT MODULUS

Particularly where vibration problems are of interest, the tangent modulus for panels in shear may be required. This is given on Data Sheet 02.03.07 and is defined by,

$$G_t = \frac{dq}{d\gamma}$$

It is of interest, therefore, to compare this Data Sheet with results obtained from T.N.2661. In fact, this latter report does not include information on the tangent modulus but, as part of the Society's revision programme, the analysis has been extended to cover tangent modulus. The appropriate equations are rather cumbersome and the computations are correspondingly laborious so this extension of the analysis has only been used to give spot checks on a graphical method for determining G_t.

In the graphical method use is made of equation (7),

$$\frac{G_t}{G} = \frac{G_s/G}{1 - \dfrac{q/q_b}{G_s/G}\dfrac{d\,(G_s/G)}{d\,(q/q_b)}} \quad . \quad . \quad (7)$$

which is derived from the definitions of G_s and G_t. Using this relation G_t/G may be determined from curves of G_s/G against q/q_b.

If, therefore, use is made of curves of G_s/G determined from T.N.2661, values of G_t/G corresponding to that analysis will be found. The result of such calculations is shown on Fig. 7 and comparison is made with the appropriate curves from Data Sheet 02.03.07. It is seen that the difference is more marked than that for G_s/G, shown on Fig. 2, and may amount to as much as 15 per cent. Again, the new analysis indicates greater stiffness than the Data Sheet.

Regarding the comparison with experimental data, it has been shown that the new analysis gives closer agreement than Data Sheet 02.03.06 for G_s/G, and thus the new analysis for G_t/G would give better agreement than Data Sheet 02.03.07.

Like the secant modulus, the tangent modulus will be affected by flange flexibility through the parameter B, and the magnitude of the effect will be of a similar order.

CONCLUSIONS

The investigation shows that the November 1941 issue of the Society's Data Sheets give somewhat lower values for the tangent and secant shear moduli than a more recent analysis. This more recent work gives closer agreement with experimental data than the Data Sheets and for this reason the revision of these sheets is under consideration.

The analysis of the effect of flange bending shows that, provided the flanges are sufficiently rigid to maintain a sensibly uniform diagonal tension field, the shear moduli are reduced by up to 10 per cent for values of the flexibility parameter of about 0·5.

ACKNOWLEDGMENT

The author wishes to acknowledge the advice given during the progress of this work both by the Society's Structures Committee and by Anthony J. Barrett, Head of the Society's Technical Department.

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Normalised Orthogonal Deflexion Functions for Beams

by

W. J. DUNCAN, C.B.E., D.Sc., F.R.S., F.R.Ae.S.

A SET of normalised orthogonal polynomial functions suitable for problems concerning doubly built-in uniform beams was given in R. & M. 2281[1]. The n^{th} function of the set was expressed linearly in terms of three Legendre polynomials as follows:—

$$\phi_n(\xi) = \tfrac{1}{4}\sqrt{(2n+1)}\,F_n(2\xi - 1), \quad . \quad . \quad (1)$$

where

$$F_n(x) = \frac{(2n-1)P_{n+2}(x) - (4n+2)P_n(x) + (2n+3)P_{n-2}(x)}{(2n-1)(2n+1)(2n+3)} \quad (2)$$

Some time ago Professor A. Erdélyi informed me that $F_n(x)$ could be expressed in the form

$$F_n(x) = \text{Constant} \times (1-x^2)^2 \frac{d^2P_n(x)}{dx^2} \quad . \quad . \quad . \quad . \quad (3)$$

I have recently verified this by a straightforward use of the recurrence relations and the differential equation satisfied by the Legendre polynomials. The final result is

$$\phi_n(\xi) = \frac{\sqrt{(2n+1)}\,(1-x^2)^2 \dfrac{d^2P_n(x)}{dx^2}}{4(n-1)n(n+1)(n+2)} \quad . \quad . \quad . \quad (4)$$

where $x = 2\xi - 1$. . . . . (5)

REFERENCE

1. DUNCAN, W. J. (1946). Normalised Orthogonal Deflexion Functions for Beams. R. & M. 2281, 1946.

Graduates' and Students' Section

SOME THOUGHTS ON NATIONAL SERVICE

THIS topic has arisen many times before in correspondence on this page and elsewhere but, although the end of National Service is in sight, it will cause much anxiety to Graduates and Students until 1960, because although the last 18-year-olds will be conscripted next year, the Ministry of Labour intends to try to satisfy the Services' needs by calling up those who have been deferred previously. Thus, extensions of deferment will be harder to get and, as I see it, the position will become stickier before National Service dies for good.

For those in design offices, the outlook is not encouraging. In one major aircraft firm, there is the prospect of one section of the design department being decimated by the call up of ex-apprentices with several years' experience. The firm is, however, able to take on raw graduates straight from university, because the present regulations recognise only a university degree as a technical qualification, and neither an H.N.C. or the A.F.R.Ae.S. Part II examination. Even so, the firm's deferment quota for university graduates has been slashed by half this year.

Clearly, the only certain way to avoid it altogether is to emigrate and then return to whatever the "Old Country" has to offer in 1970. This "unpatriotic" step will be taken by very many young scientists and engineers without their reading my seditious grumblings here.

I personally can see no point whatever in the present form of National Service for aircraft technicians, so I shall not attempt to defend it. Naturally, if anyone without an official axe to grind has a good word to say for it, they should write to the Editor. The only tales one hears are of frustration and idleness for two years.

I am sure that this feeling is shown by the attitude of most National Servicemen to those who are deferred. Their comment is usually "If you can get out of it, good luck to you!" This was shown up a few years ago when a daily newspaper conducted a readers' poll on whether or not a well-known-racing driver should "get away with it." Seventy per cent of those who replied thought he should. The odd thirty per cent probably consisted of mothers and wives who would not appreciate the matter in quite the same light.

One cannot argue honestly from pacifist standpoints, which would say that the Services are unnecessary, when one tries to avoid call-up by being employed on the design of military aircraft, but one might feel more kindly disposed to "serving the country" in the Services if they used one more efficiently.

Surely the important thing is to make the best use of the available man-power, and if the mis-employment of graduate technicians in the R.A.F. is to continue, then a very strong case for continued deferment can be made. But could not better use be made of these graduates? At present the few suitable jobs are understandably reserved for the regular officers, and the National Service graduate either gets a commission and obtains a good grounding in administration, or stays in the ranks where he sees an aspect of aircraft servicing from the practical angle. A knowledge of administration does no one any harm, nor is aircraft servicing experience without value (although the graduate has generally had enough practical experience at his parent firm). In both cases the graduate is not using his training to his or the nation's best advantage, and he is at the same time spending two years forgetting what he has learnt because he is not applying his training.

National Service implies a loss of personal liberty, and frequently involves separation from the home environment. Could not the graduate be drafted into a new Technicians Corps and posted overseas to jobs in arduous climates to which the M.o.S. finds difficulty in attracting civilian scientists and technicians? The Woomera range, Christmas Island and arctic and tropical trials are just a few suggestions which spring to mind; and if necessary the conscript could be drafted to assist aircraft firms in overseas development flying.

All this suggests that unless useful work can be found for them, graduates should be exempted by a selective call up to avoid a serious drain on the nation's trained man-power at a time when we can least afford it.—*Twenty-six year old.*

THE ANNUAL DANCE

Our popular annual function will be held this year on Friday 29th November in the Library at 4 Hamilton Place.

As yet, all the details have not been completed but they will be given in full next month on this page. However, before then, posters with the main details will appear on notice boards, where this Society's functions are usually advertised.

We shall endeavour to provide the usual good value, as and where the inflationary spiral permits!—N.K.B.

VISIT

The Road Research Laboratory. Due to unforeseen circumstances, the visit advertised last month to the Traffic and Safety Division of the Road Research Laboratory at Langley, Bucks. has had to be postponed until Saturday 7th December 1957. As described briefly last month, the visit will be very interesting to the motor-minded. All those who have already written are asked to confirm that they can manage this new date. At the time of writing, there were still some vacancies, and so it is well worthwhile writing to the Hon. Visits Secretary, Mr. R. Shepherd, 4 Heath Road, St. Albans, Herts.—N.K.B.

LECTURES

16th October. "Trends in Air Transport" by Mr. P. W. Brooks, Technical Assistant to the Chairman of British European Airways.

7th November. "The Background to Aircraft Specifications" by Mr. K. W. Clark of the Air Ministry.

THE LIBRARY

Reviews

AIRCRAFT HYDRAULICS. *Vol. II—Component Design. Edited by H. G. Conway. Chapman and Hall, London, 1957. The second of a series of text books published under the authority of the Royal Aeronautical Society. 198 pp. Diagrams. Tables. 45s.*

Volume I of this series of Royal Aeronautical Society text books, reviewed in the March 1957 Journal, dealt with hydraulic systems. This is the second of the series, and its title of "Component Design" indicates the more detailed treatment of the subject of aircraft hydraulics. Once again there is a multiplicity of authors, each an expert in his own field and thus imparting authority to this book. Credit must be given to the "actor-manager," Mr. H. G. Conway, for co-ordinating the efforts of such an all-star cast even more successfully in his second production than his first!

The book begins with a chapter on Seals by S. M. Parker, which adequately deals with the selection of suitable materials and describes many of the different types of seal arrangements. Unfortunately, the reader is given no guide whatsoever to any method of determining the size of seal required. Very rarely is there more than one dimension of a seal invariable in any application, and the benefit of the author's experience in selecting the other dimensions would have enhanced the value of the chapter to the undergraduate and young technician, for whom this book is primarily intended.

While Chapter II on Hydraulic Pumps and Motors, by E. H. Bowers, and Chapter III on Jacks, by G. Orloff, give full descriptions of basic types and well-illustrated examples of modern designs, the insufficiency of quantitative design information apparent throughout the book is evincible in the section entitled "Jack Design Principles," which deals only with dead lengths, and then only qualitatively. The statement is made that, "the type of seal used has a considerable effect on the dead length of the jack . . ." and the reader is referred to Chapter I (*vide ut supra*).

H. G. Conway writes on Selectors in Chapter IV, describing the construction and principles of operation of manually and remotely controlled types. Chapter V on Valves, by J. K. Simpson, is concerned with pump, pressure and flow control valves and other miscellaneous types for sequencing, and so on. Both these chapters are almost wholly descriptive and it is felt that more emphasis should have been given to the methods of determining valve dimensions and forces, if only by reference to published works such as those of Lee and Blackburn.

E. H. Bowers, in Chapter VI, describes Miscellaneous Hydraulic Components ranging from accumulators to pressure gauges. Naturally the limited space which can be devoted to each of the eight groups of components excuses the omission of certain additional information which is essential for the design of this miscellany. Many of them may be "off-the-shelf" items, but someone has to design them.

Chapter VII on Hydraulic Servo-Controls is remarkable for the fact that the author, F. J. Bradbury, has been able to contain himself within his allotted space on a subject which merits a whole book, and a large one at that. He has wisely restricted the content to relatively simple discussions of the fundamental mechanisms and their applications to flying and pre-selector servo-controls. There is a brief dissertation on some of the factors influencing stability and control valve design. It is a well-written introduction to the subject, the only reasonable criticism one could make being that the list of references should have been enlarged. This may seem harsh in view of the fact that out of the whole twelve references in the book, ten are appended to this chapter!

Finally, C. D. Holland writes on Pipework in Chapter VIII. My first reaction on reading this chapter was to wonder why such a mundane, though important, subject should appear in a text book for potential designers. Now I know. A subsequent fault in my domestic plumbing system found me cursing the architect, not the plumber! However, I still feel that the space given to dimensional details of A.G.S. couplings could have been put to better use in giving more vital, and less easily determined, design information in other chapters.

At first I would have wished this book to contain more quantitative information, some worked design studies, some more detailed guidance to enable the student to produce a complete, dimensional design rather than a schematic diagram. In other words, I would have liked it to be a conventional text book where one is accustomed to find formulae, useful and useless, and empirical information enabling the reader to produce a conventional design. Here, instead, one finds the experience necessary for design in a field uninhibited by convention, and which is perhaps more of an art than a science. This essential difference in approach means that the book, like any good biographical work, is of greatest value when read rather than referred to. That the contents of the book are of value is unquestionable. That the qualitative approach will appeal to the student as much as to the experienced engineer is a matter of opinion—the student's.—C. K. TROTMAN.

ELEMENTS OF GASDYNAMICS. *H. W. Liepmann and A. Roshko. Chapman and Hall, London, 1957. 439 pp. Illustrated. $11.00.*

The authors' preface states that this book was originally planned as a revised and extended version of Liepmann and Puckett's well-known "Introduction to Aerodynamics of a Compressible Fluid." Because of the large amount of new material to be included, it was later decided to write two entirely new and independent volumes, of which this is the first. It deals with the fundamentals of gas dynamics, and aims to give a sound basic knowledge of the subject, unencumbered by a large amount of detailed application. It is likely to be successful in achieving this.

In presentation and style the new book is quite similar to "Introduction to Aerodynamics of a Compressible Fluid," and an impression of its scope may be gained from a comparison with that work. It is almost twice as long as the older book. Certain sections have been greatly expanded and brought up to date, and naturally there is now much more emphasis on flight at very high speeds and under extreme conditions. The opening chapter, on thermodynamics, is now much longer and reaches a more advanced level; it contains introductory sections on the law of mass action, dissociation, condensation, and other real gas effects which are of interest in the study of hypersonics. There is a more detailed chapter on one-dimensional wave motion, and a more comprehensive

description of methods of measurement in compressible flows. Entirely new chapters deal in considerable detail with linearised theories for flow past bodies of revolution, with slender body theory, and with the similarity rules of high speed flow. The chapter on viscous effects is now more detailed, and includes simple discussions of such topics as shock wave boundary layer interactions, and the effect of dissociation on recovery temperature. Finally, the book ends with another new chapter dealing with the kinetic theory of gases, and how gas dynamics may be related to it. We are also given an extensive collection of quite well thought out problems, and a number of useful tables. A few important references are included.

This is a book which is intended mainly for students. It is not designed as a reference handbook; for example, the shock wave equations are derived, together with other material, in the course of three chapters. The authors present a sound physical background to the subject of gas dynamics without going into great detail or mathematical rigour. They often do this by introducing new or difficult topics as simple examples, which are later extended intuitively to the general case. The result is very readable, and contains a large amount of material. It can be recommended not only to students, but also to anyone who would like to take an intelligent interest in the present deluge of research reports on high speed aerodynamics. It will form a most useful starting point for later detailed study.—K. N. C. BRAY.

COMBUSTION RESEARCHES AND REVIEWS 1957. *AGARD Combustion Panel. Butterworths, London, 1957. 201 pp. Illustrated. 40s.*

Another AGARD publication on combustion: even reviewers with a fee in prospect give a sigh when they see the familiar blue AGARDOGRAPH cover. Can there really be so much new and important work in this field as to justify publication of another book?

Two hundred pages, ten papers from five countries; four on flames, two on combustion theory, two on pulse-jets, and two on physico-chemical data; some are reviews, some contain experimental data, and some give new methods of calculation. Among the experimental papers are two which will interest the student of one-dimensional flames, namely a survey of the burning velocities of mixtures of hydrogen with halogens, by Anderson, and a study of air-kerosine flames by Rappeneau. Whereas the former will help those who have been studying the theory of these flames to keep their feet on the ground, the latter will be useful to engineers who use burning-velocity data in predictions of the extinction of aircraft engine flames at high altitude. Rappeneau's most important, though expected, finding is that kerosine gives flame velocities very similar to those of simpler hydrocarbons; therefore experiments carried out on the latter give data which the engine designers can trust.

Among the review papers, that of Penner, Harshbarger and Vali gives a useful survey of ways in which the shock-tube can be used in the study of the properties, including reaction rates, of substances appearing in flames. Most of the ideas and data are not new, but their collection here may spread knowledge of them more widely. The pulse-jet papers, by Ducarme, and Bertin and Salmon, are rather slight, but contain some experimental data for which we may be grateful; too little has been published on this type of engine hitherto. Both papers throw doubt on the contention of Paul Schmidt, the inventor of the V.1, that the returning shock wave is responsible for relighting the fresh change.

Theoretical papers by Mullins and Lutz have engineering interest. The former shows how to calculate the conditions under which the vapour above liquid fuel in a tank accessible to air can support a flame; this is important in assessing the fire and explosion hazards of aircraft. The second paper presents a method of calculating the equilibrium compositions of the gas mixtures encountered in rockets; the reviewer believes that the author has shown a keen insight in his choice of variables and parameters for the thermodynamics charts presented.

It is harder to congratulate the author of the paper entitled "Theoretical Analysis of Reaction Rate Controlled Systems—Part I." This is an extensive re-hash, not always with acknowledgment, of ideas which became widely accepted in gas-turbine circles some years ago and which have since been refined and freed from many of their initial irrelevancies; the paper has not been markedly influenced by this later phase. For example, an ill-defined and misleading analysis is given of the effect of fuel diffusivity on flame stabilisation; however, the well-known work of Williams and Shipman, who brought this effect to light in 1953 and explained it adequately, is not mentioned. Let us hope that "Part II" will show more respect for the unity of the scientific literature.

In his preface the Chairman of the sponsoring AGARD panel invites constructive criticism. Here is mine. Contributors to AGARD meetings get wide and lasting publicity for their papers. These papers should therefore be subjected to at least as stringent a reviewing procedure as is adopted by learned societies and by commercial publishing houses before accepting a work for publication. Referee's criticism is needed both by workers in government establishments, who are commanded to produce a paper for the next meeting, and by academic scientists who are pressed into dashing off a "contribution." The reader would get a fairer deal too.—D. B. SPALDING.

AIRCRAFT HYDRAULIC DESIGN. *George R. Keller. Industrial Publishing Corp., London, 1957. 132 pp. Illustrated. 32s 6d.*

It is not easy to review this book constructively; it is not clear what the author's intentions were in writing it. The title and perhaps the price would imply a text book on the design of aircraft hydraulic equipment. The length of the book, however (about 20,000 words), has not permitted the author to treat any one aspect of the subject properly and the book therefore must be considered as a series of paragraphs dealing with some of the problems of modern aircraft hydraulics. From this point of view it does contain some information on high temperature problems which is useful and may not have been published before. But most of the remaining information is too isolated in context or too incomplete to be recommendable to the student; these criticisms apply particularly to some sections containing mathematical equations which seem to be included more because the author knew them than because their presence followed on a well-laid foundation.

In one astonishingly inaccurate paragraph the author states that constant delivery pump systems use relief valves "to establish the working pressure of the system and allow excess flow to return" to the tank. As every aircraft engineeer knows it is a cut-out or unloading valve which is used for this duty, and that continuous flow through a relief valve (as may be used on machine tools) cannot be accepted on aircraft, because of heat generation.

As this book has been published in the United States using American references and illustrations, the British engineer may find something of interest in it; it is probable that its value to American engineers, already familiar with much of it, is much less.—H. G. CONWAY.

AVIATION FACTS AND FIGURES 1957. *Published by Aircraft Industries Association of America Inc. (American Aviation Publications, Washington 5 D.C.) 120 pp. Illustrated. $1.00.*

This is one of the annuals that has bred into a perennial and to the appearance of which, one librarian at least, looks forward each year. It solves many of his provoking inquiries on how much? how many? and "over the past few years—?" Considering the implication of the title, there is a surprising amount of text, but tables are there too, on such subjects as:—

Personnel in the U.S.A.F., 1912 to date; Certificated Civil and Student pilots, 1927 to date; Aircraft in Service on world airlines; The ten leading Passenger Transport Companies; U.K. employment and production in the aircraft manufacturing industry; and so on.—F.H.S.

ROYAL AIR FORCE. *John W. R. Taylor. Ian Allen, London, 1957. 37 pp. Illustrated. 2s. 6d.*

The publishers have changed the size of this latest addition to the "abc" series. It is slightly larger and will, therefore, not be so readily recognisable on the bookstalls. Mr. Taylor has done a creditable job of cramming into 37 pages as much encouragement as possible to young men to continue to join the R.A.F.

Résumés, rather than chapters, are given on the R.A.F. organisation and training, Aircraft markings and methods of procurement. The badges and insignia of rank are spread over two pages and this looks suspiciously like space-filling. The bulk of the book is devoted to the familiar Taylor-material "Aircraft of the Royal Air Force."—F.H.S.

WOMAN PILOT. *"Jackie" Moggridge. Michael Joseph, London, 1957. 235 pp. 16s.*

This is the autobiography of "Jackie" Moggridge, who is already well-known to the public as the receiver of the King's Commendation for ferrying a greater number of aircraft during the war than any other pilot of either sex.

She humorously tells of her early attempts to qualify for an "A" Licence, and displays in the telling an amazing modesty, when one realises her later accomplishments. One sees the gradual change from the "shy, pious and rather priggish" teen-ager into the wise and mature aviatrix, who was to become the source of envy and inspiration to thousands of young women the world over.

The description of the long months she spent in Aeronautical College and later in the W.A.A.F. could very easily have seemed dull to the layman, but the author has succeeded in maintaining interest with an imaginative style and many shrewd comments. No reader could fail to sympathise with her struggles to be accepted on an equal basis with her male colleagues.

This is a book which should be read by all who believe a woman's place is in the home. Mrs. Moggridge has managed to combine a happy marriage with a most demanding career, and has done so in spite of her husband's complete lack of interest in aviation.—MURIEL HANNING-LEE.

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SOCIETY OF AIRCRAFT MATERIALS AND PROCESS ENGINEERS: PAPERS PRESENTED TO CONFERENCE ON ADHESIVE BONDED STRUCTURES FOR AIRCRAFT JAN./FEB. 1957

2. THERMAL INSULATING PROPERTIES AND CHARACTERISTICS OF LOW DENSITY PLASTIC CORE MATERIAL. A. C. Rawuka.
4. ADHESIVE BONDING OF STAINLESS STEEL FOR HIGH TEMPERATURE EXPOSURE. F. J. Riel.
6. HOW DIFFERING ADHESIVE PHYSICAL PROPERTIES CAN INFLUENCE HONEYCOMB SANDWICH PERFORMANCE. W. C. O'Leary and A. F. Martin.
21. PROCESS CONTROL TESTS FOR BONDED STRUCTURES—EVALUATION OF. J. W. Woodhouse.
22. PROCESS CONTROL AND PRODUCTION PRACTICES IN SANDWICH PRODUCTION. E. Hagberg.
23. QUALITY CONTROL TECHNIQUES IN HONEYCOMB CORE PRODUCTION. R. D. Johnston.
24. A DISCUSSION OF METAL BONDING CONTROLS. H. Hendriksen.
38. A REINFORCED PLASTIC SANDWICH MISSILE STRUCTURE DESIGNED FOR USE AT 650°F. L. O. North.
42. METAL-TO-METAL BONDING AND FLIGHT SAFETY. C. Thomas.
43. SWEDISH AERONAUTICAL RESEARCH ON METALLIC HONEYCOMB SANDWICH CONSTRUCTION. B. R. Noton.

* *Items marked thus are for reference use only.*

Reports

AERODYNAMICS

BOUNDARY LAYER—*see also* THERMO-AERODYNAMICS
INTERNAL FLOW
WING AND AEROFOILS
COMPRESSIBLE FLOW

*The effect of heat transfer on interactions involving laminar boundary layers. K. N. C. Bray. C.P.*339. 1957.

A theory is developed to allow for heat transfer effects, in interactions between laminar boundary layers and shock waves, and on boundary layer separation. As a simple example of the use of this theory, the effect of wall temperature on the Mach number change to cause separation of a laminar boundary layer has been found, for large Reynolds numbers. Suggestions are made for further calculations, based on the method.—(1.1.1.4 × 1.9.1).

Lift hysteresis at stall as an unsteady boundary-layer phenomenon. F. K. Moore. N.A.C.A. Report 1291. 1956.

With the problem of rotating compressor stall in view, an unsteady boundary layer calculation is made to find the unsteady increment of lift of a stalled elliptic aerofoil in a flow of time-dependent incidence. The analysis indicates a counter-clockwise hysteresis loop of the lift curve, the various forms of which are discussed.—(1.1.1.1 × 1.10.1.1).

Méthodes d'exploration dynamique et thermique et étude de la couche-limite laminaire compressible de la plaque plane. M. Plan. Pubs. Sc. et Tech. No. 331. 1957.

Etude systématique des mesures locales de vitesse et de température dans la couche-limite. Revue des méthodes de détermination théorique de ces grandeurs, choix d'une méthode particulière, valeurs numériques obtenues. Comparaison des déterminations expérimentale et théorique pour la couche-limite laminaire; influence de l'épaisseur du bord d'attaque. Recueil des valeurs numériques des propriétés physiques de l'air intervenant dans les calculs, variations avec la température.—(1.1.1.4).

Estimation of compressible boundary-layer growth over insulated surfaces with pressure gradient. G. W. Englert. N.A.C.A. T.N. 4022. *June* 1957.

Incompressible boundary layer theory is extended using a modified transformation to include compressible boundary layer development over insulated surfaces. The variation of turbulent shear stress with Mach number of the bounding potential stream is considered using a reference-temperature method. Profile parameter as well as momentum thickness can still be evaluated by simple quadratures using this method.—(1.1).

Compressible laminar boundary layer over a yawed infinite cylinder with heat transfer and arbitrary Prandtl number. E. Reshotko. N.A.C.A. T.N. 3986. *June* 1957.

Numerical solutions to the equations for the compressible laminar boundary layer have been obtained for the flow in the stagnation region of an infinite cylinder in yaw for large free-stream Mach numbers and large yaw angles. The wall temperature was varied from zero to twice the stream stagnation temperature. Solutions are given for Prandtl numbers of 1 and 0·7.—(1.1.1 × 1.9.1).

COMPRESSIBLE FLOW—*see also* WINGS AND AEROFOILS

*Some integrated volume properties in linearized flow and their connection with drag reduction at supersonic speeds. Z. O. Bleviss. Douglas Report No. SM-*19469. *February* 1956.

In some recent linearised studies of the drag of generalised (non-planar) aircraft configurations in supersonic flow it was assumed that the aircraft configurations could be represented by singularity distributions (elements of lift, side force and volume) and generalised distributions were studied. It is obviously necessary to relate the distribution of the singularities to the geometry of the aircraft configuration. One aspect of this problem, namely the relation of the volume of a linearised body to the strength of the singularities, initiated the present investigation. In studying this relationship some simple general local and integrated volume properties of linearised flow are obtained.—(1.2.3).

On the influence of the geometry of slender bodies of revolution and delta wings on their drag and pressure distribution at transonic speeds. F. Keune and K. Oswatitsch. K.T.H. Aero T.N. 42. 1956.

With the help of the area rule the flow around delta wings and swallow-tailed wings in the transonic range is referred to the flow around (equivalent) bodies of revolution. Since the critical Mach numbers lie quite near unity, the transonic properties can be determined by the calculation of the critical Mach numbers, the calculation in the linearised ranges, and at a Mach number of one. For the latter the "parabolic method" is used and applied together with a transonic method of characteristics to various bodies. The lower critical Mach number is determined with sufficient accuracy by the linear theory. For the upper critical Mach number the formula of Oswatitsch-Sjodin is used. After general considerations a geometrical system of bodies of revolution is introduced. The suitability of this system is checked by applying to the linear range. Some examples are given for wings equivalent to the bodies of revolution considered. The sonic flow is calculated and the results are given in a most general form by the transonic similarity.—(1.2.2).

*Supersonic drag calculations for a cylindrical shell wing of semi-circular cross-section combined with a central body of revolution. B. J. Beane and B. M. Ryan. Douglas Report No. SM-*22627. *October* 1956.

A semi-circular wing with a body of revolution on the axis is studied to find the wave and the vortex drag for various chordwise lift distributions and for three values of a parameter describing the wing geometry $\beta R/(c/2)$. Using the least drag, together with the vortex and skin friction drags, the maximum lift to drag ratio for each wing geometry is computed.—(1.2.3).

The effect of a spike protruding in front of a bluff body at supersonic speeds. D. Beastall and J. Turner. R. & M. 3007. 1957.

Wind tunnel tests at $M=1{\cdot}5$, $1{\cdot}6$ and $1{\cdot}8$ are described in which the effect of mounting a stem, with different nose pieces, on the forward face of a bluff-nosed body is studied. The drag on the front face of the body was derived from pressure measurements for different projections of the stem. Schlieren equipment was used and several interesting phenomena observed in the flow are discussed.—(1.2.3).

One-dimensional treatment of non-uniform flow. R. D. Tyler. R. & M. 2991. 1957.

The errors likely to arise when the simple one-dimensional flow methods are applied to a circular section duct in which a boundary layer exists are illustrated. Graphical results are presented, for the case of a one-seventh power law boundary layer velocity profile, showing the ratio of the true mean values calculated with allowance for the boundary layer, to quantities derived from the simple one-dimensional calculation. Various boundary layer thicknesses and a range of Mach numbers are dealt with. Specifically three examples are worked out in detail, with different selections of the four independent variables, the selections being chosen to cover problems of common interest. General equations are presented for use in cases not covered by these examples. They are analogous to the one-dimensional equations, and give ratios of mean flow quantities to their sonic values, as functions of Mach number and correction factors, graphically presented, which depend on the velocity distribution. As a further illustration of possible application of the theory, the correction factors may be used for the calculation of momentum flux or kinetic energy flux from the mean velocity and mean density.—(1.2 × 1.1 × 1.5.1).

NOTE.—*The figures in parenthesis at the end of each Summary are for office use only.*

The influence of pressure ratio and divergence angle on the shock position in two dimensional, over-expanded, convergent-divergent nozzles. P. F. Ashwood and G. W. Crosse. C.P. 327. 1957.

Tests have been made to determine the influence of divergence angle and applied pressure ratio on the shock position in over-expanded, Laval-type, convergent-divergent nozzles. The tests covered a range of design pressure ratios between 3·5 and 17·0 and included divergence angles of from 5° to 40°. An empirical expression has been derived which enables the limiting pressure ratio at which the nozzle just runs full to be calculated and curves are presented from which the shock position within the nozzle can be obtained. The variation with design pressure ratio of the maximum pressure ratio obtainable across the exit shock has been determined and the results compared with those predicted using a semi-empirical formula derived from shock wave-boundary layer interaction experiments. Good agreement is obtained. In the absence of a suitable drying plant humid air had to be used for all the tests. The effects of this on the measured pressure distributions are discussed.—(1.2.3.2).

The effects of liquefaction of air on the pressure distribution and forces on a cone-cylinder body at $M=5{\cdot}0$. W. T. Strike and W. N. MacDermott. Astia Doc. No. AD. 123509.

An investigation of the influence of liquefaction of air on the aerodynamic characteristics of a cone-cylinder model configuration at a Mach number of 5·0 was conducted in a 12 in. supersonic wind tunnel. Supply temperatures ranged from 130°F below saturation value to 60°F above saturation value, at a constant supply pressure of four atmospheres. Effects of liquefaction on the nozzle calibration as well as on the model pressure distribution and overall integrated forces are determined.—(1.2.3).

Performance of a Mach 4 *axially symmetric nozzle designed to operate at* 40 *microns Hg in the UTIA low density wind tunnel. A. K. Sreekanth. UTIA T.N. No.* 10. *September* 1956.

The performance of a Mach 4 axi-symmetric nozzle designed to operate at a test chamber pressure of 40 microns of mercury is investigated in the UTIA low density wind tunnel both at the design condition and for other values. Additional investigations are made on the effect of the stagnation temperature and the pressure in the open jet test chamber on the flow.—(1.2.3).

CONTROL SURFACES—*see also* WINGS AND AEROFOILS

Control surfaces restrained by viscous friction as a means of damping aircraft oscillations. W. J. G. Pinsker. R. & M. 2962. 1957.

A method is described of controlling the phase of the free motion of control surfaces by viscous friction and geared masses. The merits of various arrangements are discussed and formulae for the determination of optimum conditions are derived. The conclusions are illustrated by numerical examples.—(1.3 × 1.8).

Investigations at low speeds of deflectors and spoilers as gust alleviators on a model of the Bell X-5 airplane with 35° *swept wings and on a high-aspect-ratio* 35° *swept-wing-fuselage model. D. R. Croom and J. K. Huffman. N.A.C.A. T.N.* 4057. *June* 1957.—(1.3 × 1.6.3.).

INTERNAL FLOW—*see also* COMPRESSIBLE FLOW

The internal flow problem in axi-symmetric supersonic flow. J. J. Mahony. ARL Report A/101. *January* 1957.

A scheme of approximate solution is presented for the supersonic flow in circular ducts of slowly varying cross-section for the cases when conventional linearised theory fails. A simple method is obtained for converting the singularities of the linearised solution into the uniformly valid form. The problem of an expansive discontinuity in the slope of the wall of the duct is treated and the details of the flow clarified.—(1.5.1).

Radial focussing in supersonic flows in ducts. J. J. Mahony. ARL Note A/160. *April* 1957.

The problem of the radial focusing of small disturbances in a supersonic flow in a duct of circular cross-section is considered.—(1.5.1).

Measurements of the thrust produced by convergent-divergent nozzles at pressure ratios up to 20. *P. F. Ashwood et al. C.P.* 326. 1957.

Tests have been made on a series of convergent-divergent nozzles having an included divergence angle of 20° to obtain performance data at pressure ratios applicable to jet engines operating in the flight speed range $M=2$ to $M=3$. A new test technique has been used which, by utilising the ejector action of the jet from the test nozzle to reduce the exit pressure, enables the range of pressure ratios obtainable across the nozzle to be expanded threefold. This technique, which has proved simple to apply and control, is described in detail.—(1.5.1).

Comportement de grilles d'aubes de compresseur de courbure et d'allongement différents. C. Rouffignac. O.N.E.R.A. Note Technique No. 39. 1957.

Les résultats de l'etude expérimentale en grilles rectilignes d'aubes de compresseur sont présentés.—(1.5.2.3).

Investigation of a short-annular-diffuser configuration utilizing suction as a means of boundary-layer control. S. W. Wilbur and J. T. Higginbotham. N.A.C.A. T.N. 3996. *June* 1957.

A straight outer-wall annular diffuser having a centre body length of one-half the outer-body diameter and an area ratio of 1·9:1 is investigated for mean inlet flow angles of 0° and 19·5° with an inlet Mach number of 0·26 to determine the effect of area suction applied on the inner wall. The entrance shape, number, and location of the openings through which the air was removed are varied. The auxiliary air flow is varied from 0 to approximately 4 per cent of the main-stream air flow.—(1.5.1 × 1.1.5.1).

Nonuniformities in shock-tube flow due to unsteady-boundary-layer action. H. Mirels and W. H. Braun. N.A.C.A. T.N. 4021. *May* 1957.

The boundary layer along the walls of a shock tube induces pressure and velocity gradients within the core of potential flow. These non-uniformities are evaluated for shock tubes in which the boundary layer is thin relative to the tube diameter and is either wholly laminar or wholly turbulent. The hot gas region between the shock wave and the contact surface is considered. Both the axial distributions at any instant and the temporal distributions at any axial position within this region are found. Numerical computations are presented for an air-air shock tube.—(1.5.1.4 × 1.1).

Design and experimental evaluation of a light-weight turbine-wheel assembly. W. C. Morgan and R. H. Kemp. N.A.C.A. T.N. 4023. *June* 1957.

Light-weight design concepts are developed and tested in a full-scale turbine wheel with a tip diameter of 34·3 in. and a tip speed of 1,190 ft. per sec. The rotor consisted of two thin discs attached to each other by spacers and bolts. Each disc was contoured for optimum stress distribution. The turbine blades were hollow castings with an integral serrated root. The total weight of the wheel, exclusive of the shafts and bolts, was 121 lb.—(1.5.3 × 27.1).

Thermodynamic study of a Roots compressor as a source of high-temperature air. C. B. Cohen et al. N.A.C.A. T.N. 4025. *June* 1957.

Thermodynamic analysis of a Roots blower indicates that leakage of hot air from the high- to the low-pressure side produces a large preheating effect on the supply air. The resulting outlet temperatures are extremely sensitive to heat losses in the cycle.—(1.5.2 × 34).

LOADS—*see* CONTROL SURFACES
STABILITY AND CONTROL
WINGS AND AEROFOILS

PERFORMANCE ESTIMATION

The effect of a wind gradient on the rate of climb of an aircraft. W. A. Mair. R. & M. 2953. 1957.

The behaviour of an aircraft climbing in the presence of a wind gradient is analysed by a method similar to that in R. & M. 379, but with fewer simplying assumptions.—(1.7).

STABILITY AND CONTROL—*see also* CONTROL SURFACES

*Measurements of some low speed oscillatory stability derivatives of the AGARD wing model D. J. Yff. NLL-TM F.*195. 1956.

Some results of dynamic stability measurements of the AGARD wing model D are presented, together with a derivation of the equation of motion and a description of the test technique and the apparatus. The tests have been conducted in a low speed wind tunnel at values of the reduced frequency ranging from 0·015 to 0·040 and at Reynolds number of 400,000-760,000.—(1.8.1 × 2).

Charts for estimating the effects of short-period stability characteristics on airplane vertical-acceleration and pitch-angle response in continuous atmospheric turbulence. K. G. Pratt and F. V. Bennett. N.A.C.A. T.N. 3992. *June* 1957.

Charts are presented from which the root-mean-square vertical acceleration and pitch angle in dimensionless form can be estimated for values of four other dimensionless parameters that describe the aeroplane short-period stability characteristics and the scale of atmospheric turbulence.—(1.8.2.1 × 1.6.3).

THERMO-AERODYNAMICS—*see also* BOUNDARY LAYER

A review of theoretical work relevant to the problem of heat transfer effects on laminar separation. G. E. Gadd. C.P. 331. 1957.

The results of various theories concerning the effects of heat transfer on laminar separation are briefly discussed. All the theories predict that wall temperature has an important influence on separation conditions.—(1.9.1 × 1.1.1).

Contribution a l'étude de la convection forcée de la chaleur sur des parois rugueuses. G. Brunello. Pubs. Sc. et Tech. No. 332. 1957.

Etude de l'influence de la rugosité dans les phénomènes de convection forcée de la chaleur.—(1.9.1 × 34.3.2).

Heat transfer in a laminar boundary layer at Mach 2·5 from a surface having a temperature distribution. B. N. Pridmore-Brown. UTIA Report No. 45. *February* 1957.

Heat transfer measurements are made on the outer surface of a hollow cylinder with a sharp leading edge at a Mach number of 2·5. The surface of the model was divided into half-inch segments, the temperature of which could be controlled and measured individually. Heat transfer results for various surface temperature distributions are compared with the theory of Chapman and Rubesin. Boundary layer pitot traverses are made with a rectangular mouthed probe.—(1.9.1 × 1.1.1.4).

WINGS AND AEROFOILS—*see also* BOUNDARY LAYER

An analysis of available data on the local aerodynamic centres of aerofoils in two- and three-dimensional flow. A. S. Taylor. R. & M. 3000. 1957.

Existing information on the behaviour of the local aerodynamic centres of aerofoils is reviewed, with a view to exposing the more important gaps in our knowledge, and indicating the lines along which future research might most profitably be directed. Starting with the two-dimensional aerofoil in incompressible, viscous flow, for which aerodynamic centre position may be correlated with lift slope, the report examines the behaviour of the two-dimensional aerodynamic centre in incompressible flow. Considering next the locus of aerodynamic centres for wings of finite aspect ratio, the various incompressible potential flow theories and their extension to the subsonic and transonic regimes of compressible flow are discussed, and the published results of calculations by various investigators collected in a series of figures. Brief mention is made of supersonic theory. Other figures present experimental data.—(1.10).

*Examples of calculation of minimum supersonic drag due to lift by solution of two-dimensional potential problem: elliptical-planform wings and an approximate delta-planform wing. M. E. Graham. Douglas Report No. SM-*22754. *March* 1957.

Two examples are presented of the use of two-dimensional potential flow solutions to obtain minimum drag due to lift of three-dimensional planar wings. The method of relating the two-dimensional potential problem to the three-dimensional drag minimisation problem for spatial as well as planar wing systems can be developed from combined flow field concepts. For the first example, the minimum drag of unyawed and yawed elliptical-planform wings is obtained. For the second example, the minimum drag of a wing with a planform very nearly that of a sonic-edge delta wing is obtained.—(1.10.1.1).

*A series of aerofoils designed to develop exceptionally large lift coefficients when boundary layer control by blowing is employed. D. G. Hurley and P. R. Skeat. A.R.L. Report A.*102. *March* 1957.

A series of aerofoils has been designed by an extension of classical free stream-line theory to develop very large lift coefficients when a blowing slot is used to suppress boundary layer separation. One of the aerofoils which has a thickness/chord ratio of 0·62, a trailing edge angle of 90° and a design lift coefficient of 9·69 has been investigated in detail.—(1.10.1.1 × 1.1.4).

Second-order subsonic airfoil theory including edge effects. M. D. Van Dyke. N.A.C.A. Report 1274. 1956.

Several recent advances are combined into a unified second-order theory for aerofoil sections in subsonic flow. The method includes correction of the thin-wing approximation near stagnation points. Solutions for various profiles are compared with results of experiment and other theories. A computing scheme is given for treating any profile numerically.—(1.10.1.1 × 1.2.1.1).

Perforated sheets as the porous material for a suction-flap application. R. E. Dannenberg et al. N.A.C.A. T.N. 4038. *May* 1957.

Two-dimensional tests are made of a N.A.C.A. 0006 aerofoil with area suction applied to a porous region on a 0·3-chord trailing-edge flap deflected 50°.—(1.10.2.1 × 1.3.4 × 1.1.5.1).

Investigation at transonic speeds of deflectors and spoilers as gust alleviators on a 35° *swept wing. Transonic-bump method. D. R. Croom and J. K. Huffman. N.A.C.A. T.N.* 4006. *June* 1957.

An investigation is made in the Langley high-speed 7 ft. by 10 ft. tunnel by means of the transonic-bump method to determine the gust alleviation capabilities of a deflector control and a spoiler-deflector control when mounted on a 35° swept semi-span wing having N.A.C.A. 65A006 aerofoil sections.—(1.10.2.2 × 1.3.4 × 1.6.3).

Experimental investigation of the aerodynamics of a wing in a slipstream. M. Brenckmann. UTIA T.N. No. 11. *April* 1957.

An experimental study of a wing in a propeller slipstream is made to determine the distribution of the lift increase due to slipstream at different angles of attack of the wing and at different free stream-slipstream velocity ratios, the results being intended as an evaluation basis for different theoretical treatments of this problem.—(1.10.2.1).

TESTING AND INSTRUMENTS

The operation of the N.P.L. 18 *in.* × 14 *in. wind tunnel in the transonic speed range. I. M. Hall. C.P.* 338. 1957.

A brief description of the slotted liners used is given together with the power requirements and some flow surveys. Some observations are made on wall interference on a half-model of a swept wing tested in the wind tunnel.—(1.12.1.2).

Strain gauge balances for wind tunnels. An outline of practice in the United Kingdom. J. R. Anderson. AGARD Report 5. *February* 1956.

A brief survey of developments in strain gauge balance

practice in the United Kingdom is made, with notes on the various types of balance used, their design and fabrication. Descriptions of twelve particular strain gauge balances are included.—(1.12.6.2).

A wide-frequency-range air-jet shaker. R. W. Herr. N.A.C.A. T.N. 4060. *June* 1957.

A description of a simple air-jet shaker is presented. Its force can be calibrated statically and appears to be constant with frequency. It is relatively easy to use, and it has essentially massless characteristics. This shaker is applied to define the unstable branch of a frequency-response curve obtained for a non-linear spring with a single degree of freedom.—(1.12 × 2).

AEROELASTICITY

See AERODYNAMICS—TESTING AND INSTRUMENTS *and* STABILITY AND CONTROL

STRUCTURES —TESTING *and* THEORY AND ANALYSIS

DESIGN AND CONSTRUCTION

Some considerations of hysteresis effects on tire motion and wheel shimmy. R. F. Smiley. N.A.C.A. T.N. 4001. *June* 1957.

A theoretical study is made of the influence of tyre hysteresis effects on the rolling motion and wheel shimmy of landing gears. The results of this study indicate that hysteresis forces and moments have a noticeable secondary influence on the landing-gear rolling behaviour. Comparisons of the available experimental data with the corresponding theoretical predictions provide a fair confirmation of the theory.—(4.2.2.3 × 33.1).

Some effects of valve friction and stick friction on control quality in a helicopter with hydraulic-power control systems. B. P. Brown and J. P. Reeder. N.A.C.A. T.N. 4004. *May* 1957.

Flight tests have been made in a helicopter with hydraulic-power control systems to investigate the effect of various ratios of servo-valve friction and stick friction, both with and without feel devices. The tests showed that, when an objectionable amount of valve friction is present, some benefits can be gained by introducing stick friction. The addition of feel devices resulted in an overall improvement in the control systems.—(4.4).

AIRCRAFT OPERATION

L'accroissement de la precision a l'atterrissage par l'emploi d'indicateur d'incidence. P. Lecomte. AGARD Report 30. *February* 1956.

Le problème de l'atterrissage sur des pistes relativement courtes pose divers problèmes techniques. La note ci-dessous donne quelques indications sur l'emploi de l'indicateur d'incidence pour l'approche et sur les avantages qu'on peut en attendre au point de vue : - de la diminution des vitesses d'approche - de l'augmentation de la précision d'impact. Deux examples de réalisations pratiques sont fournies.—(5).

Utilisation des terrains sommairement amenages. S. Lefebvre. AGARD Report 74. *August* 1956.

Cette note essaye de faire la synthèse des enseignements tirés d'essais divers concernant l'étude des problèmes liés à l'emploi des avions sur des terrains sommairement aménagés. Cette synthèse est présentée en deux parties, après un bref rappel des difficultés particulières rencontrées dans l'examen des qualités de roulement des avions sur les terrains non homogènes que sont les pistes en terre.—(5).

United States army experience in operations from unprepared fields. J. F. Denhart. AGARD Report 80. *August* 1956.

This non-technical presentation is based on the use of light aircraft to support the Army avaition mission. Although it may not be related to the use of heavier and faster aircraft which normally support the Air Force mission, some of the requirements and methods considered by the Army for the accomplishment of its mission may be applicable to high-performance aircraft.—(5).

Philosophy of safety in the supersonic age. N. J. Hoff. AGARD Report 87. *August* 1956.

Present and past principles governing the proof of the structural strength of aircraft are reviewed. It is suggested that the safety of the aeroplane in the presence of rarely occurring unusually large loads, of fatigue, and of creep can be judged on a common basis only if the probability of the failure of the structure is accepted as the safety criterion.—(5.3).

Appraisal of the hazards of friction-spark ignition of aircraft crash fires. J. A. Campbell. N.A.C.A. T.N. 4024. *May* 1957.

A study was made to determine if common aircraft metals produce friction sparks capable of igniting combustibles that might be spilled in an aeroplane crash. Samples of aluminium, titanium, magnesium, chrome-molybdenum steel, and stainless steel were dragged over both concrete and asphalt runways while a combustible mixture was sprayed around the sample. No ignitions occurred from the sliding aluminium; but the other metals produced friction sparks that ignited the combustibles.—(5.3 × 34.1.1).

FLIGHT TESTING

Preliminary report on a gust alleviator investigation on a Lancaster aircraft. J. Zbrozek et al. R. & M. 2972. 1957.

The investigation of gust alleviator effectiveness is limited to an analysis of statistical measurements of e.g. accelerations.—(13).

HYDRODYNAMICS

An empirical analysis of the planing lift characteristics of rectangular flat-plates and wedges. P. Ward Brown. R. & M. 2998. 1957.

The fluid flow about planing plates and wedges is described briefly and discussed, and on the basis of theoretical and physical considerations of this flow empirical formulae are presented for the lift developed by these planing surfaces. The formulae are mutually compatible and cover the whole range of planing of zero and finite deadrise surfaces including the chine-dry and chine-wet conditions. The lift formulae are extensively compared with experimental data, over a range of trim angles from 2 to 30 degrees and deadrise angles from 0 to 40 degrees, and good agreement obtained.—(17.2).

Recherches théoriques et expérimentales sur les mouvements des liquides pesants avec surface libre. A. S. Apté. Pub. Sc. et Tech No. 333. 1957.

Etude théorique de la configuration limite de l'onde solitaire. Théorie de la génération des harmoniques de la houle par un batteur trois-barres. Calcul des seiches provoquées dans un ouvrage portuaire de forme rectangulaire et de profondeur constante, par une houle plane. Contrôle expérimental.—(17.1).

Etude de quelques problèmes sur les ondes liquides de gravité G. Brillouet. Pub. Sc. et Tech. No. 329. 1957.

Recherche de la fonction harmonique $\phi(x, y)$ la plus générale de deux variables x et y, qui vérifie les conditions suivantes: $\phi(x, y)$ n'est singulière que pour $x=y=0$, $\phi_y-\phi=0$ sur Ox, $d\phi/dn=0$ sur $x \sin \alpha+y \cos \alpha=0$, lorsque: $\alpha=p\pi/2q$ (p impaire avec q et inférieur à $2q$). Application au problème des ondes liquides de gravité qui constituent une représentation approchée du mouvement de la mer au voisinage des côtes.—(17).

Microdébitmètre a perte de charge D.F.G. F. Debeauvais, R. Feidt et C. Gebel. Pub. Sc. et Tech. N.T. 64. 1957.

Principe et justification d'un microdébitmètre à perte charge. Théorie de l'instrument avec ses corrections hydrodynamiques et thermiques, conduisant à la définition d'une courbe d'étalonnage unique à échelle fonctionnelle des températures. Etalonnage de l'instrument, basé sur des mesures absolues de précision, déterminant les coefficients hydrodynamiques et les corrections de température. Réalisation de l'instrument, comprenant le microdébitmètre proprement dit avec son manomètre de mesure d'une part, l'abaque d'utilisation avec son appareil de lecture d'autre part.—(17.1).

MATERIALS

See also STRUCTURES—THEORY AND ANALYSIS

The ageing characteristics of ternary aluminium-zinc-magnesium alloys. I. J. Polmear. A.R.L. Report Met. 20. *January* 1957.

A systematic study has been made of the ageing characteristics of a wide range of ternary aluminium-zinc-magnesium alloys with basic zinc contents of 4, 6 and 8 per cent zinc, and magnesium contents within the range of 0-3 per cent. Hardness-Ageing Time curves were determined at temperature intervals between —20° and 240°C and tests made for periods of up to two years duration.—(21.2.2).

Conductibilité électrique aux très basses températures des métaux de très haute pureté et application aux phénomènes de recristallisation. M. Caron. Pub. Sc. et Tech. No. 328. 1957.

Nous avons mis au point une méthode physique de classement et d'identification d'aluminiums et de fers de haute pureté obtenus par de nouvelles méthodes de purification en particulier par la méthode de la zone fondue. Ce classement est conforme à celui que l'on obtient par les méthodes d'analyse microchimique et radiochimique. Nous avons été conduit par la suite à des recherches métallographiques et chimiques sur les aluminiums de très haute pureté.—(21.1).

Influence of crucible materials on high-temperature properties of vacuum-melted nickel-chromium-cobalt alloy. R. F. Decker et al. N.A.C.A. T.N. 4049. *June* 1957.

A study of the effect of induction-vacuum-melting procedure on the high temperature properties of a titanium-and-aluminium-hardened nickel-base alloy revealed that a major variable was the type of ceramic used as a crucible. Reactions between the melt and magnesia or zirconia crucibles apparently increased high-temperature properties by introducing small amounts of boron or zirconium into the melts. Heats melted in alumina crucibles had relatively low rupture life and ductility at 1,600°F and cracked during hot-working as a result of deriving no boron or zirconium from the crucible.—(21.2.2).

MATHEMATICS

Les séries chronologiques et la théorie du hasard. M. Kiveliovitch et J. Vialar. Pubs Sc. et Tech. T.N. 65. 1957.

L'étude des séries chronologiques conduit les auteurs à construire des tests du hasard, les uns qualitatifs, les autres quantitatifs. Ces tests permettent de décider si une série d'observations peut être considérée comme purement aléatoire ou présente une certaine organisation. De nombreux exemples d'application tirés des différentes branches de la science sont traités.—(22.1).

Extrapolation techniques applied to matrix methods in neutron diffusion problems. R. R. McCready. N.A.C.A. Report 1283. 1956.

An iterative scheme is developed for the matrix solution of the type of characteristic value problem arising from homogeneous linear equations. The scheme is shown to minimise a suitable form at each step. Extrapolation techniques for speeding convergence are developed and refined. An example from nuclear reactor theory is presented.—(22.1).

MECHANICAL ENGINEERING

A generalisation of the Nyquist stability criterion with particular reference to phasing error. R. H. Merson. C.P. 330. 1957.

The effect of phasing error on the stability of a two-dimensional linear servo-mechanism is considered and it is shown that the system will be stable if the phase margin at the cut-off frequency exceeds the phasing error. The more general case of a number of identical servos with cross-coupling is investigated and a generalisation of the Nyquist criterion for stability is formulated.—(23.3).

METEOROLOGY

The variation of gust frequency with gust velocity and altitude. N. I. Bullen. C.P. 324. 1957.

Information on atmospheric turbulence obtained from counting accelerometer records is examined and relations giving the variation of gust frequency with gust velocity and altitude are obtained. The results are summarised in a form convenient for use in estimating the fatigue life of an aircraft.—(24 × 31.3.1).

Results of a flight investigation on clear air turbulence at low altitude using a Meteor Mk. 7 *aircraft. R. M. Allan. C.P.* 329. 1957.

Runs were made over a given route at heights below 1,000 ft. above ground and the accelerations due to gusts were recorded by a continuous trace accelerometer. The results of all the runs showed that, on the average a 1*g* gust or greater occurred every 7·06 minutes. Comparision with other information showed that the results fitted in with existing data. There were considerably more positive than negative gusts, the factor being 18 at 1·2*g* or greater. It would appear that for the area in which the flights were made, turbulence near the ground is not isotropic. A consideration of meteorological factors showed that on this route wind is far more powerful in producing turbulence at low altitude than the effect of sun.—(24).

Atmospheric turbulence encountered by Hermes aircraft. J. R. Heath-Smith. C.P. 334. 1957.

Acceleration records were obtained of the turbulence encountered in 417,000 miles of operational flying on routes from London to East and West Africa. It is shown that turbulence decreases with increasing altitude. A result of the pilot's discretion in choice of altitude and course is that the gust frequency encountered during most of the cruise is about one third of the average atmospheric gust frequency. Between 9,500 ft. and 14,500 ft. gusts were encountered four times as frequently over East and West African sectors as over European sectors.—(24).

Statistical study of aircraft icing probabilities at the 700- *and* 500-*millibar levels over ocean areas in the northern hemisphere. P. J. Perkins et al. N.A.C.A. T.N.* 3984. *May* 1957.

A statistical study is made of icing observations reported from weather reconnaissance aircraft. Wide areas of the Pacific, Atlantic, and Arctic Oceans were surveyed at fixed flight levels of 18,000 ft. and 10,000 ft. Icing statistics presented include the relative frequencies of the occurrence of icing, the estimated probability of flight in icing, and the relation of these probabilities to the frequencies of flight in clouds and cloud temperatures.—(24).

POWER PLANTS

See AERODYNAMICS—INTERNAL FLOW

PROPELLERS

An analysis of once-per-revolution oscillating aerodynamic thrust loads on single-rotation propellers on tractor airplanes at zero yaw. V. L. Rogallo et al. N.A.C.A. Report 1295. 1956.

A simplified procedure is shown for calculating the once per revolution oscillating aerodynamic thrust loads on propellers of tractor aeroplanes at zero yaw. The only flow field information required is a knowledge of the upflow angles at the horizontal centre line of the propeller disc. Methods are presented whereby these angles may be determined by theoretical procedures.—(29.1).

REFERENCE LITERATURE

Sources of translation of aeronautical interest in NATO countries. L. J. H. Haylor and R. A. Kennedy. AGARD Report 56. *August* 1956.

Government departments, industrial research organisations, universities, professional institutions and private organisations making and/or indexing translations of aeronautical interest in the NATO countries are included. The main

indexing centres are: N.R.C. (Canada); C.N.R.S. and S.D.I.T. (France); Aslib (U.K.); S.L.A.-John Crerar Library and S.T.C. (U.S.A)—(30.4).

FATIGUE

See METEOROLOGY
STRUCTURES—THEORY AND ANALYSIS *and* TESTING

STRUCTURES

LOADS—*see also* DESIGN AND CONSTRUCTION

A summary of ground-loads statistics. J. R. Westfall et al. N.A.C.A. T.N. 4008. *May* 1957.

The more important statistical data obtained by the N.A.C.A. on the subject of ground loads is summarised. The information presented relates primarily to landing-impact and taxying loads; some limited data are also presented on braking friction. A bibliography is included.—(33.1).

Loads implications of gust-alleviation systems. W. H. Phillips N.A.C.A. T.N. 4056. *June* 1957.

A brief review is presented of the basic methods of gust alleviation. Some results obtained in flight tests of a gust alleviation system are included.—(33.1.2).

THEORY AND ANALYSIS

Strengths of Avdel light alloy blind rivets in DTD 546B sheet. B. M. Lempriere. CoA Note 63. *April* 1957.

Results are given comparing the double- and single-shear strengths of joints in D.T.D. 546B light alloy sheets using Avdel blind rivets. The double-shear proof and ultimate strengths were found to be appreciably better than the single-shear strengths over a useful range of sheet thickness/rivet diameter ratio. The effect of manufacturing tolerance was examined and found to be important. Consideration is given to the physical behaviour of the joints which is seen to differ in three ranges of thickness/diameter ratio.—(33.2.4.13.4).

*Torsional stiffness and combined loading tests on the "Olympic" glider fuselage. S. W. Gee. A.R.L. Note SM.*237. *October* 1957.

Tests were conducted on a monocoque wooden structure, representing a glider rear fuselage, to observe its behaviour under pure torsion and to determine the ultimate strength under combined torsion, shear and bending.—(33.2.3.3).

The analysis of the structural behavior of guyed antenna masts under wind and ice loading. G. J. Schott, F. R. Thurston and P. J. Pocock. N.R.C. Report MER-1. 1956.

An analytic procedure is described for computing the distribution of bending moment, shear, axial load and deformation in the mast, and the loads in the guy wires, for a 3-level, 4-way guyed antenna mast erection subjected to wind load, ice accretion and loads applied by an antenna installation at the head of the mast. The computational procedure is described explicitly with examples, and permits the immediate use of normal computational aids. Part II of the report provides wind tunnel test results on a number of mast designs with simulated ice accretion.—(33.2.3.4).

Recent research on the determination of natural modes and frequencies of aircraft wing structures. J. M. Hedgepeth. AGARD Report 37. *April* 1956.

Some recent theoretical and experimental studies of the vibrations of aircraft wing structure are discussed. The importance of various secondary effects is assessed and method of including them are outlined. Experimental values for natural frequencies of box-beam specimens with theoretical calculations are compared and it is shown that, particularly for the higher modes, good agreement needs the inclusion of secondary effects.—(33.2.4.1.10×33.2.3.2×2).

A method for solving dynamic problems of modern transonic and supersonic wings. L. Broglio. AGARD Report 38. *April* 1956.

A method is presented for the general solution of dynamic problems of delta, swept or crescent wings, or those of a more general type. The first part gives a method for the determination of frequencies and modes of vibration, with the influence coefficients and mass distribution assumed known. In the second part the calculation of the influence functions is discussed and, in the third, the methods of the previous two parts are applied to practical problems. The Appendix shows how to determine the influence functions in specific simplified cases.—(33.2.3.2).

Effect of interaction on landing-gear behavior and dynamic loads in a flexible airplane structure. F. E. Cook and B. Milwitzky. N.A.C.A. Report 1278. 1956.

The effects of interaction between a landing gear and a flexible aeroplane structure on the behaviour of the landing gear and the loads in the structure have been studied by treating the equations of motion of the aeroplane and the landing gear as a coupled system where the landing gear is considered to have non-linear characteristics. The effects of neglecting interaction and using the landing-gear forcing function for a rigid body in a dynamic analysis of a flexible airframe are also discussed.—(33.2.3.0).

Theoretical investigation of flutter of two-dimensional flat panels with one surface exposed to supersonic potential flow. H. C. Nelson and H. J. Cunningham. N.A.C.A. Report 1280. 1956.

A Rayleigh type analysis is used to treat the flutter of a two-dimensional flat panel supported at its leading and trailing edges and subjected to a middle-plane tensile force, with a supersonic stream over its upper surface and still air below. Numerical results in the form of stability boundaries are presented for determining some effects of number of modes, Mach number, densities of the supersonic stream and still air, panel mass and stiffness, edge fixity, structural damping, and middle-plane tensile force. Comparison is made with a few experimental results.—(33.2.4.5.10).

Some experimental studies of panel flutter at Mach number 1·3. *M. A. Sylvester and J. E. Baker. N.A.C.A. T.N.* 3914. *February* 1957.

Experimental studies of panel flutter using thin metal plates were conducted at a Mach number of 1·3 to study the effects of some structural parameters on the flutter characteristics. The effects of tensile forces and buckling were studied on panels clamped front and rear, in addition to initially buckled panels clamped on all four edges. Panel flutter was obtained under controlled laboratory conditions and it was found that tensile forces, shortening the panels, and increasing the bending stiffness were effective means for eliminating flutter. Buckled panels were more susceptible to flutter than unbuckled panels. No apparent systematic trends in the flutter modes or frequencies could be observed.—(33.2.4.5.7).

Experimental influence coefficients and vibration modes of built-up 45° *delta-wing specimen. E. E. Kordes et al. N.A.C.A. T.N.* 3999. *May* 1957.

Experimental influence coefficients and vibration modes and frequencies of a built-up 45° delta-wing specimen are presented. The symmetrical and anti-symmetrical static influence coefficients were obtained on a three-point support. The first 10 vibration modes and frequencies were obtained for an essentially free-free condition. A detailed description of the structural properties of the specimen is also given.—(33.2.3.2).

A variational theorem for creep with applications to plates and columns. J. L. Sanders et al. N.A.C.A. T.N. 4003. *May* 1957.

A variational theorem is presented for a body undergoing creep. Solutions to problems of creep bending and creep collapse of plates, columns, beams, and shells can be obtained by means of the direct methods of the calculus of variations in conjunction with the stated theorem. The application of the theorem is illustrated for plates and columns by the solution of two sample problems.—(33.2.4.0.9 ×21.2.0).

Some research results on sandwich structures. M. S. Anderson and R. G. Updegraff. N.A.C.A. T.N. 4009. *June* 1957.

The results of compressive-buckling tests of steel sandwich plates are given, and the significant parameters which affect the strength of the plates are discussed. The various types of sandwich construction are shown to be comparable on a weight-strength basis with conventional high-strength aluminium-alloy construction.—(33.2.4.6.6).